POLARISATION OF LIGHT

Fig. 1. Fig. 2.

Fig. 3. Fig. 4.
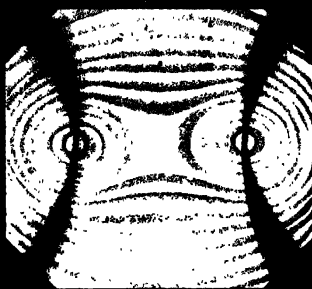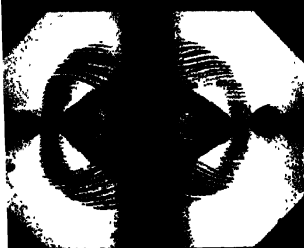
Fig. 5. Fig. 6.
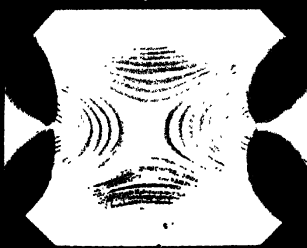
Fig. 7
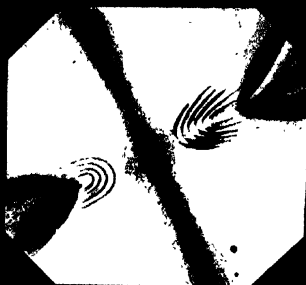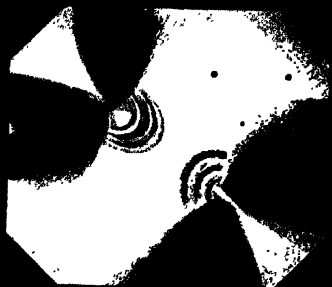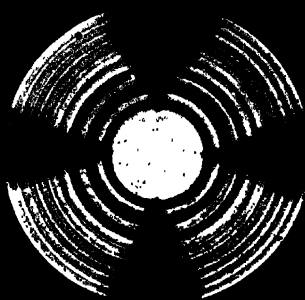
Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

POLARISATION OF LIGHT.

BY

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PREFACE.

THE FOLLOWING PAGES contain the substance of Lectures delivered at various times to my workpeople, and constitute a talk rather than a treatise on Polarised Light. If a perusal of them should induce some to read, and others to write, more fully on the subject, this provisional sketch will have served its purpose. For such utility as it may possess I have to thank Professors Tyndall, G. G. Stokes, and Maskelyne, Sir C. Wheatstone, Professor W. G. Adams, and others whose suggestions I have more or less consciously adopted; and last, but not least, those of my audience who, by patient attention to the Lectures, have encouraged me to pursue the study of this beautiful branch of Optics.

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CONTENTS.

Description of the Frontispiece.

POLARISATION OF LIGHT.

CHAPTER I.

METHODS OF POLARISATION.

LIGHT is said to be polarised when it presents certain peculiarities, hereafter to be described, which it is not generally found to possess. These peculiarities, although very varied in their manifestations, have one feature in common, viz., that they cannot be detected by the unassisted eye; consequently, special instrumental means are required for their investigation.

The origin and meaning of the term polarisation will be better understood when some of the phenomena have been witnessed or described than beforehand, and I therefore postpone, for the present, an explanation of it.

The subject of polarisation may be approached by either of two roads, the experimental or the theoretical. The theoretical method, which proceeds upon the principles of the Wave Theory of Light, is remarkably complete and explicit; so much so that it not only connects together many very diversified phenomena.

but even, in some cases, has suggested actual predic-
tion. But inasmuch as the theory without experimental
facts would be little better than a study of harmony
without practical music, it will be best to begin with
experiment.

It was stated above that certain instrumental means
were requisite for detecting polarisation. Now there
are various processes, some occurring in the ordinary
course of natural phenomena, others due to instrumental
appliances, whereby a ray of light may be brought
into the condition in question, or "polarised;" and it
is a fact both curious in itself and important in its
applications, that any one of these processes (not
necessarily the same as that used for polarising) may
be used also as a means of examining whether the ray
be in that condition or not. This latter process is
called "analysation." When two instruments, whether
of the same or of different kinds, are used, they are
called respectively the "polariser" and the "analyser;"
and the two together are included under the general
name of "polariscope."

The four principal processes by means of which a
ray of light may be polarised are, reflexion, ordinary
refraction, double refraction, and scattering by small
*particles. These methods will be considered in order;
but before doing so, it will be convenient to describe*
the phenomena of polarisation as exhibited by some
instrument tolerably simple in its action, and of easy
manipulation. For such a purpose a plate of crystal
called tourmalin will perhaps serve better than any
other to begin with.

Tourmalin is a crystal of which there are several

varieties, differing only in colour.   Very dark specimens
generally answer the purpose well, excepting that it is
difficult to cut them thin enough to transmit much
light.   Red, brown, or green specimens are usually
employed ; the blue are, for the most part, optically
unsuitable.   Some white, or nearly white, specimens
are very good, and may be cut into thicker plates
without loss of light..

If we take a plate of tourmalin cut parallel to a
particular direction within the crystal called the optic
axis (the nature and properties of which will be more
particularly explained hereafter), and interpose it in
the path of a beam of light at right angles to the
direction of the beam, the only effect perceptible to
the unassisted eye will be a slight colouring of the
light after transmission, in consequence of the natural
tint of the particular piece of crystal.   But if we
examine the transmitted beam by a second similar
plate of tourmalin, placed parallel to the former, the
following effects will be observed.   When the two
plates are similarly placed, *i.e.* as if they formed one
and the same block of crystal, or, as it is technically
expressed, with their optic axes parallel, we shall per-
ceive only, as before, the colouring of the light due to
the tints of the two plates.   *But if either of the plates
be then turned round in its own plane, so as always to
remain perpendicular to the beam, the light will be
observed to fade gradually, until, when the moving
plate has been turned through a right angle, the light
becomes completely extinguished.   If the turning be
continued beyond the right angle, the light will begin
to revive ; and* when a second right angle has been

completed, the light will be as bright as at the outset.
In Figs. 1 and 2, *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, represent the two
plates ; in Fig. 1 the two plates are supposed to be in
the first position ; in Fig. 2 the plate *e*, *f*, *g*, *h*, has
been turned through a right angle. Of the parts
which overlap, the shading in Fig. 1 represents the
deepened colour due to the double thickness of the
crystal ; in Fig. 2 it indicates the complete extinction
of the light. The same alternation of brightness and
extinction will continue for every right angle through
which the moving plate is turned. .Now *it* is to be
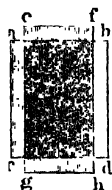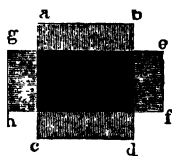
FIG. 1.   FIG. 2.

observed that this alternation depends only upon the
angle through which one of the crystals has been
turned, or, as it is usually stated, upon the relative
angular position of the two crystals. Either of them
may be turned, and in either direction, and the same
sequence of effect will always be produced. But if
the pair of plates be turned round bodily together, no
change in the brightness of the light will be made.
It follows, therefore, that a ray of ordinary light
possesses the same properties all round ; or, as it may
be described in more technical language, a ray of
ordinary light is symmetrical in respect of its properties

about its own direction. On the other hand, a ray
of light, after traversing a plate of tourmalin, has
properties similar, it is true, on sides, or in directions,
diametrically opposite to one another, but dissimilar
on intermediate sides or directions. The properties
in question vary, in faot, from one angular direction
to another, and pass through all their phases, or an
entire period, in every angle of 180°. This directional
character of the properties of the ray, on account of
its analogy (rather loose, perhaps) to the directional
character of a magnet or an electric current, suggested
the idea of polarity ; and hence the condition in which
the ray was found to be was called polarisation.

Having so far anticipated the regular order of
things on the experimental side of the subject, it will
perhaps be worth while to make a similar anticipation
on the side of theory. It is considered as established
that light is due to the vibrations of an elastic medium,
which, in the absence of any better name, is called
ether. The ether is understood to pervade all space
and all matter, although its motions are affected in
different ways by the molecules of the various media
which it permeates. The vibrations producing the
sensation of light take place in planes perpendicular
to the direction of the ray. The paths or orbits of
the various vibrating ethereal molecules may be of any
form consistent with the mechanical constitution of
the ether ; but on the suppositions usually made, and
none simpler have been suggested, the only forms
possible are the straight line, the circle, and the
ellipse. But in ordinary light the orbits at different
points of the ray are not all similarly situated ; and

although there is reason to believe that in general the orbits of a considerable number of consecutive molecules may be similarly situated, yet in a finite portion of the ray there are a sufficient number of variations of situation to prevent any preponderance of average direction.

This being assumed, the process of polarisation is understood to be the bringing of all the orbits throughout the entire ray into similar positions. And in the case of the tourmalin plate the orbits are all reduced to straight lines, which consequently lie in one and the same plane. For this reason the polarisation produced by tourmalin, as well as by most other crystals, is called rectilinear, or, more commonly, plane polarisation. This property of tourmalin may also be expressed by saying that it permits only rectilinear vibrations parallel to a particular direction, determined by its own internal structure, to traverse it.

Adopting this view of polarisation as effected by a plate of tourmalin, it would be interesting to ascertain the exact direction of the vibrations.[1] And a simple experiment will go far to satisfy us on that point. The argument, as now stated at least, is perhaps based upon general considerations rather than upon strict mechanical proof; but the experimental evidence is so strong that it should not be denied a place here. Suppose for a moment that the tourmalin be so placed that the direction of vibration lies either in or perpen-

[1] It is still an open question whether the vibrations of polarised light are executed in, or perpendicular to, the plane of polarisation. Opinion nclines to the latter view, which is adopted throughout this work.

dicular to the plane of incidence (that is, the plane
containing the incident ray, and perpendicular to the
surface on which it falls at the point of incidence) ;
then it is natural to expect that vibrations executed
in the plane of incidence will be far more affected by
a change in the angle of incidence than those perpen-
dicular to that plane.   In fact, the angle between the
direction of the vibrations and the surface upon which
they impinge will, in the first case, vary with the angle
of incidence ; but in the second case it will remain
unchanged.   •

In Figs. 3 and 4, *n o* represents the ray of light ;
the arrow the direction of vibration ; *a, b, c, d, a', b',
c', d'*, the plate in two positions, turned in the first
instance about the direction of vibration, in the second
about a line perpendicular to it.

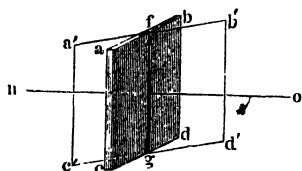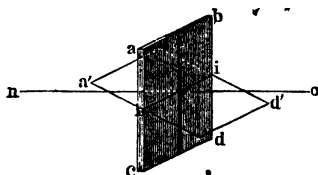FIG. 3.   FIG. 4.

Dismissing, then, the former supposition, and
supposing that nothing whatever is known about the
direction of vibration, then, if all possible directions
be taken in succession as pivots about which to tilt
or turn the second tourmalin, it will be found that for
one direction the intensity of the light diminishes
more rapidly with an increase of tilting (or, what is

the same thing, with an increase of the angle of inci-
dence) than for any other. And, further, that for a
direction at right angles to the first, the intensity of
light diminishes less than for any other; while for
intermediate directions the diminution of intensity is
intermediate to those above mentioned. In accord-
ance, therefore, with what was said before, we may
conclude that the vibrations are parallel to the line
or pivot about which the plate was turned when the
diminution of light was least.

Secondly, polarisation may be effected by reflexion.
If light reflected from the surface of almost any,
except metallic, bodies be examined with a plate of
tourmalin, it will in general be found to show traces
of polarisation ; that is to say, if the plate of tourmalin
be caused to revolve in its own plane, and the reflected
rays be viewed through it, then in certain positions
of the plate the reflected light will appear less bright
than in others. If the angle at which the original
rays fall upon the reflecting surface be varied, it will
be found that the amount of alteration in brightness
of the light seen through the revolving tourmalin (or
analyser) will also vary. This fact may also be ex-
pressed thus : In polarisation by reflexion, the degree
of polarisation, or the amount of polarised light in
the reflected rays, varies with the angle of incidence
on the reflecting surface. But at a particular angle,
called on that account the polarising angle, the polari-
sation will be a maximum. This angle (usually
measured between the incident ray and the perpendi-
cular to the reflecting surface) is not the same for all

substances; in fact, it varies with their refractive
power according to a peculiar law, which, when stated
on the technical language of science, may be thus
enunciated : The tangent of the polarising angle is
equal to the refractive index. Simple geometrical
considerations, combined with the usual expressions
for the laws of reflexion and refraction, will show that
this relation between the polarising angle and the
refractive index may be also expressed in the follow-
ing way : If light be incident at the polarising angle,
the reflected and refracted rays will be at right angles
to one another.

In Fig. 5, *s i* represents the incident, *i f* the re-
flected, and *i r* the refracted ray. Then *s i* will be
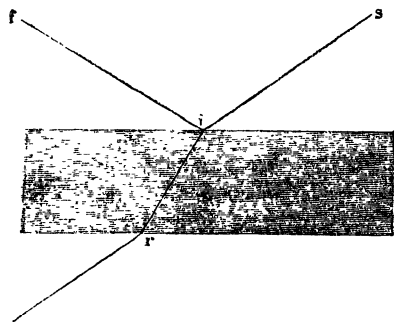
FIG. 5.

incident at the polarising angle when the angle *f i r*
is a right angle.

An apparatus devised by Professor Tyndall for
experimentally demonstrating the laws of reflexion
and refraction is admirably adapted for verifying this

law.   The following description is quoted from his
Lectures on Light : " A shallow circular vessel R I G
(Fig. 6), with a glass face, half filled with water ren-
dered barely turbid by the admixture of a little milk
or the precipitation of a little mastic, is placed upon
its edge, with its glass face vertical.   By means of a
small plane reflector M, and through a slit I in the
hoop surrounding the vessel, a beam of light is ad-
mitted in any required direction."   If a little smoke
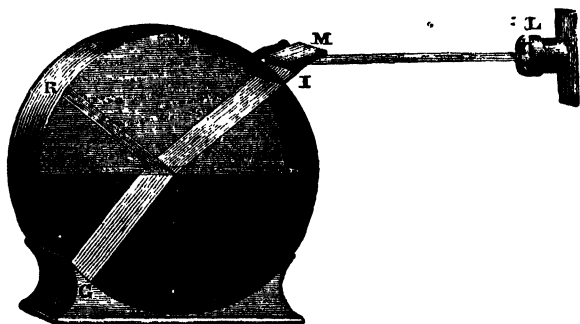
FIG. 6.

be thrown into the space above the water, the paths
of the incident, the reflected and the refracted beams,
will all be visible.   If, then, the direction of the inci-
dent beam be so adjusted that the reflected and the
refracted beams are at right angles to one another,
and a Nicol's prism be interposed in the path of the
incident beam, it will be found that, by bringing the
vibrations alternately into and perpendicular to the
plane of incidence, we shall render the intensity of

the reflected and refracted rays alternately a minimum. Thus much for the verification of the law. But not only so ; if we take different fluids, and for each of them in succession adjust the incident beam in the same manner, we shall only have to read off the angle of incidence in order to ascertain the polarising angle of the fluid under examination.

The polarising angle for glass is 54° 35', that for water somewhat less.

Returning to the general question of polarisation by reflexion, and calling to mind a principle stated above, viz. that any process which will serve for polarising will serve also for analysing, it follows that we may replace the analys- ing tourmalin by a plate of glass or almost any other non-metallic substance. If two similar reflecting plates be used, one for polarising, the other for analysing the light, they must be so placed as to receive the rays which fall upon them at the polar- ising angle. Thus, in Fig. 7, let *a b* be the incident, and *b c* the reflected ray at the first plate ; *b c* the inci- dent, and *c d* the reflected ray, at the second plate ;

FIG. 7.

then the ray *b c* will be polarised more or less, accord- ing to the angle of incidence, at *b*, and will be analysed at *c*. If the second plate be then turned round the ray
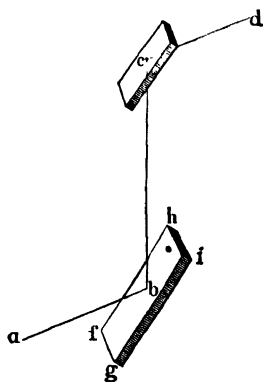
b c reflected from the first plate as an axis, it will be
·found that at two positions of rotation (first, when the
plates are parallel, and, secondly, when one of them
has been turned through 180°) the light reflected
from the second plate is brightest, and at two posi-
tions at right angles to the former the reflected ray
is least bright.   The degree of dimness at the two
positions last mentioned will depend upon the accu-
racy with which the reflecting plates have been
adjusted to the polarising angle ; and when this has
been completely effected, the light will be altogether
extinguished.

   Suppose, now, that the reflecting substance be, as
in the case of glass, transparent.   Then it will not be
surprising if, when the reflected ray is polarised, the
refracted ray should also exhibit traces of polarisa-
*tion.   And, in fact, every ray of ordinary light inci-
dent at any angle upon a transparent plate is partly*
reflected and partly refracted ; the reflected ray is par-
tially polarised, and so also is the refracted ray.   This
being so, if, instead of a single plate, we use a series
of plates ·placed one behind the other, the light will
be reflected in all possible ways from the two *n*
surfaces, where *n* represents the number of the plates
used.   The phenomenon of these reflexions is, there-
fore, rather complicated ; but the modifications due
⁻to the additional plates do not materially alter the
proportion of polarised to unpolarised light.   It is,
however, otherwise with the refracted rays.   The rays ·
transmitted by the first plate enter the, second in a
state of partial polarisation, and by a second trans-
mission undergo a further degree of polarisation.   If

this process be continued by having a sufficient num-
ber of plates, the ray finally emergent may have any
degree of polarisation required.[1] And it is worthy
of remark that, in proportion as the rays become
more and more polarised, so does a less and less
quantity of light become reflected from the surfaces
of the plates ; and, consequently, except in so far as
light is absorbed by actual transmission through the
substance of the plates, the emergent ray suffers
less and less diminution of intensity by each addi-
tional plate; so that when a certain number has
been attained, the intensity received by the eye, or on
a screen, is practically unaffected by increasing their
number.

Fig. 8 is a general representation of such a pile
of plates viewed edge-ways. The plates are secured
in a brass frame, and the whole supported on a stand.

The experiment described above, in which the
rays reflected from the pile of glass plates are extin-
guished by the analyser when in one position, while
those which have been transmitted are extinguished
when the analyser is in a position at right angles to
the former, shows that the vibrations of the reflected
and refracted rays, so far as they become polarised,
are at right angles to one another. And further, if
these rays be severally examined with a plate of
tourmalin, it will be found that the vibrations of the
reflected ray are executed in a direction perpendi-

---

[1] Plates of the thinnest description are the best. Two or three give
good effects; but if the surfaces lie parallel and the glass be highly
transparent, the number may be advantageously increased to 10 or
even 12.

cular to the plane of incidence, and those of the
refracted ray in a direction parallel to that plane.
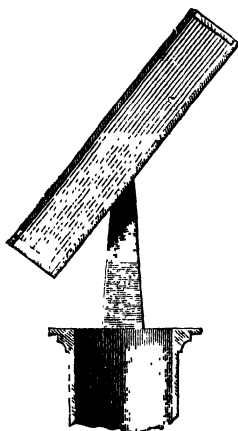
FIG. 8.

The same general reasoning as that used in the
case of tourmalin plates will serve, if not as actual
proof, at all events as illustration in this case.   Thus,
suppose that a ray whose vibrations are perpendicular
to the plane of incidence—that is, parallel to the re-
flecting surface—fall upon a plate of glass, then there
is no apparent reason why a change in the angle of
incidence should modify the reflexion and refraction,
so far as they depend directly upon the direction of
the vibrations.   The vibrations cannot undergo any
change of direction on one side rather than on the
other by incidence on a surface to which they are
parallel, and will consequently remain parallel to

themselves even when the incidence has taken place. And since the reflected and refracted rays both lie in the plane of incidence, the vibrations (which are perpendicular to that plane, and consequently to every line in it) will fulfil the optical condition of being perpendicular to the rays in question. But if the vibrations of the incident ray take place in the plane of incidence, it is difficult to conceive that the results of reflexion and refraction should be unaffected by a change in the angle of incidence. The general theory of the reflexion and refraction of polarised light was first established by Fresnel, who based it upon the four following suppositions :—

(1.) That the ether, to the vibrations of which light is supposed to be due (as will be more fully explained in Chapter III.), is regarded as perfectly elastic, so that the whole of the motion taking place at the source of light is transmitted without loss throughout the ray. This appears to be substantially true in transparent media; but in proportion as a substance is more opaque, so do its molecules take up part of the motion of the ether, and convert the light into heat (the principle of *vis viva*).

(2.) That, in passing from one medium to another, although the velocity and extent of the motion may change, yet its character is not altered (the principle of continuity).

(3.) That any change, in velocity or in extent, due to the passage from one medium to another, takes place immediately at the surface of separation ; and that such change is maintained subsequently.

(4.) That while the elasticity of the ether in dif-

ferent bodies remains the same, its density may differ.

By means of these suppositions relations were established between the intensity of the reflected and refracted rays on the one hand, and the angles of re-flexion and refraction on the other, from which many phenomena previously known only as experimental facts were deduced as consequences. Of these one should be mentioned here, viz. that in the case of vibrations in the plane of incidence, if the ray be incident at such an angle that the reflected and re-fracted rays are perpendicular to one another, there can be no reflected ray.

CHAPTER II.

DOUBLE REFRACTION—POLARISCOPES.

WE next come to the subject of polarisation by double refraction. There are a large number of crystals which have the property of generally dividing every ray which passes through them into two. But the extent of separation of the two rays varies with the direction of the incident ray in reference to the natural figure of the crystal. In every double refracting crystal there is at least one, and in many there are two, directions in which no such separation takes place. These directions are called optic axes. The relations between the forms of crystals and their optic axes, and optical properties arising therefrom, will be explained later.

Of such crystals Iceland spar is the most notable instance. If we take a block of such spar split into its natural shape, a rhombohedron, Fig. 9, and for convenience cut off the blunt angles by planes perpendicular to the line joining them, *a b*, it will be seen that a ray of light transmitted perpendicularly to these planes, that is, parallel to the line joining the blunt angles, is not divided. In fact, the image either of

the aperture of the lantern projected on a screen, or of an object seen by the eye in the direction in question, appears single, as if passed through a block of glass. The direction in question (viz. the line *a b* itself, and
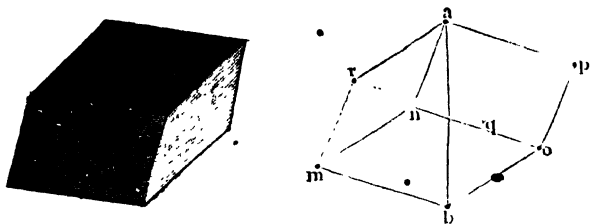
FIG. 9.

all lines passing through any part of the crystal parallel to *a b*) is called the optic axis of the crystal. If, however, the crystal be tilted out of this position in any direction, it will be seen, by the appearance of two images instead of one, that the rays are divided into two. The angular divergence of the two sets of rays, or, what comes to the same thing, the separation of the two images, depends upon the angle through which the crystal has been turned ; or, as it may also be expressed, upon the angle between the directions of the incident ray and the optic axis of the crystal. When this angle amounts to a right angle the separation is at its greatest ; and if the crystal be still further turned, the images begin to come together again until, when it has turned through another right angle, they coincide.

This process of separation, or doubling the rays, is called double refraction. And the following experi-

ment will show that one set of rays follows the ordinary law of refraction, while the other follows a different law. The image produced by the first set of rays is, in consequence, called the ordinary, and that produced by the second the extraordinary image. Let us now take a sphere of Iceland spar, which will act upon the rays issuing from the lamp as a powerful lens. In every position in which it is placed it produces two images on the screen ; but in one position the two images are concentric, and differ only in this, that one is larger than the other. The direction in which the light is then passing is that of the optic axis. If then we suppose the curvature of the sphere to be gradually diminished, we should find the difference of the sizes of the two images, as well as the absolute size of both, diminish ; until, when the surfaces of the lens became flat, the difference would vanish, and the two images would perfectly coincide.

This difference in the size of the images shows, moreover, a very important property of double refracting crystals. The amount of refraction produced by a transparent medium standing in air depends, as is well known, upon the velocity with which a ray of light traverses the medium compared with that with which it traverses air. The smaller the velocity in the medium, the greater the refraction. The greater the refraction, the greater the magnifying power of a lens constructed of that medium. Hence in the two concentric images we can at once point to the system of rays which has traversed the crystal at a lower velocity than the other.

Let us now turn the crystal round into some other position, so that the direction of the optic axis shall no longer coincide with that of the rays from the lamp or from the object. During this process one of the images, the larger, remains stationary, as would be the case with the single image, if we had used a sphere of glass. This, therefore, is the ordinary image. The other shifts about, separating itself from the first, until the crystal has been turned through half a right angle, and then drawing back again until the crystal has swept round through a complete right angle. This is, consequently, the extraordinary image.

It will be noticed that when the sphere has been turned through a right angle, the extraordinary image is no longer circular, but elliptical, and that the major axis of the ellipse lies in the direction in which the motion has taken place, that is, perpendicular to the axis about which the sphere has been turned. This is due to the fact, shown above, that the nearer the direction of the incident rays to that of the optic axis, the less the divergence between the ordinary and the extraordinary rays. The distortion of the image when the sphere has turned through half a right angle is due to the difference of angles between the optic axis and the rays which enter the crystal on one side and on the other of the central ray of the beam coming from the lamp.

That the rays forming each of the images are polarised, and that the direction of their polarisation is different, is easily shown by interposing a plate of tourmalin or other polarising instrument between the

lamp and the sphere of spar. When the central ray
from the lamp traverses the sphere in a direction
parallel to the optic axis, the tourmalin shows that
the polarisation in the ordinary image is radial, as
will be more fully explained in a later chapter of this
work. In fact, the intensity of the light in the ring,
whereby the ordinary exceeds the extraordinary
image, will be seen to be greatest at two opposite
points, and least at two opposite points each $90°$
from the former two ; while in intermediate parts the
light exhibits gradations from the maximum to the
minimum intensity. For other positions of the sphere,
the polarisation is still further complicated ; so that
the character of the polarisation of the two images is
better studied by using flat instead of curved surfaces
for separating the rays.

For the purpose in question there is, perhaps, no
better instrument than the double-image prism. This
consists of a combination of two prisms, one of
Iceland spar, so cut that the optic axis is parallel to
the refracting edge ; the other of glass, and usually
having a refracting angle equal to that of the spar.
The rays passing through the crystal prism being
perpendicular to the optic axis, undergo the greatest
separation possible. And the chromatic dispersion
caused by that prism is usually corrected or neutral-
ised entirely in the case of the extraordinary, and
nearly so in that of the ordinary ray, by the glass
prism which is placed in a.reverse position. In this
arrangement the extraordinary image occupies the
centre of the field, and remains fixed when the double-

image prism is made to revolve in a plane perpendi-
cular to the incident rays; while the ordinary image
is diverted to a distance from the centre, and revolves
in a circle about that centre, when the prism revolves.
Other dispositions of the double-image prism are also
made for particular purposes; e.g. in which neither
image is central, and in which the chromatic disper-
sion of both images is partially corrected.

If the nature of the light in the two images thus
formed be examined by any polarising instrument, it
will be found that it is polarised in both cases; and
that the vibrations in the one image are always per-
pendicular to those in the other. And in particular
the vibrations in the extraordinary image are parallel,
and those in the ordinary are perpendicular to the
optic axis.

On these principles polarising and analysing in-
struments have been constructed by various combina-
tions of wedges or prisms of Iceland spar, the details
of which it is not necessary to describe in full. But
the general problem, and object proposed, in all of
them has been to cause such a separation of ordinary
and extraordinary rays, that one set of rays may, by
reflexion or other methods, be further diverted and
afterwards thrown altogether out of the field of
view. This done, we have a single beam of com-
pletely polarised light and a single image produced
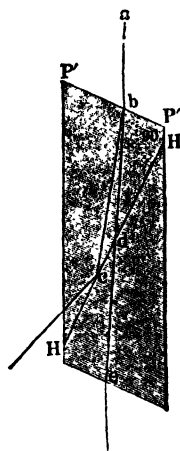from it.

One such instrument, however, the Nicol's prism,
on account of its great utility and its very extensive
use, deserves description. A rhombohedron of Iceland

spar double of its natural length is taken (see Fig.
10) ; and one of its terminal faces P, which naturally
makes an angle of 71° with the blunt edges K, is cut
off obliquely so as to give the new
face, say P′ (not given in the figure),
an inclination of 68°
to the edges K. The
whole block is then
divided into two by a
cut through the angle
E′ in a direction at
right angles to the
new face P′ ; the
faces of this cut are
then carefully polished,
and cemented toge-
ther again in their
original position with
Canada balsam. Fig. 11 represents
a section of such a prism made by
a plane passing through the edges K (Fig. 10). A
ray entering as *a b* is divided into two, viz. *b c* the
ordinary, and *b d* the extraordinary. But the refrac-
tive index of the Canada balsam is 1·54, *i.e.* inter-
mediate between that of the spar for the ordinary
(1·65) and the extraordinary (the minimum value of
which is 1·48) rays respectively ; and in virtue of
this the ordinary ray undergoes total reflexion at the
surface of the balsam, while the extraordinary passes
through and. emerges ultimately parallel to the
incident ray. • Fig. 12 shows an end view of a Nicol's

FIG. 10.

FIG. 11.

prism, PP representing the plane of polarisation of the emergent polarised ray.
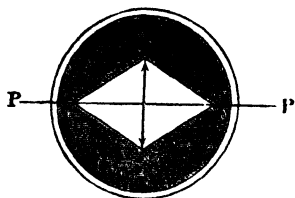

FIG. 12.

   Two such instruments, when used together, are respectively called the "polariser" and the "analyser," on account of the purposes to which they are put. These, when placed in the path of a beam of light, give rise to the following phenomena, which are, in fact, merely a reproduction in a simplified form of what has gone before.

   When polariser and analyser are placed in front of one another with their shorter diagonals parallel, that is, when the vibrations in the image transmitted by the one are parallel to those in the image transmitted by the other, the light will be projected on the screen exactly as if only one instrument existed. If, however, one instrument, say the analyser, be turned round, the light will be seen to fade in the same way as in the case of the tourmalin plates ; until, when it has been turned through a right angle, or, as it is usually expressed, when the polariser and analyser are crossed, the light is totally extinguished.

In the complete apparatus or polariscope, we may incorporate any system of lenses, so that we may make use of either parallel or convergent light, and finally focus the image produced upon the screen or upon the retina. At present we shall speak only of the phenomena of colour produced by crystal plates in a parallel beam of polarised light—chromatic polarisation, as it is called, with parallel light.

Various forms of polariscopes have been devised, whereof the three described below, and in Chapters V. and VIII., may be regarded as the most important.

Fig. 13 is an elevation of one of them. When used in its simplest form, the frame F carries a plate of black glass, which is capable of revolving about pivots in the uprights. The positions of the source of light and of the frame must be adjusted so that the plate will receive the incident light at the polarising angle, and reflect it in the direction of the eye-piece, which contains a Nicol or other analyser. The objects to be examined are to be placed on the diaphragm E.

This instrument may be converted into another form, due to Norremberg, by placing a silvered mirror horizontally at H. The plate of black glass must be removed from the frame F, and a plate of transparent glass substituted for it, which must be so inclined that the light falling upon it shall be reflected at the polarising angle perpendicularly towards the horizontal mirror. The object may be placed on the diaphragm E as before. But it may also be placed on the dia-

phragm D below the polarising plate F, and in that
case the eye will receive the polarised ray reflected
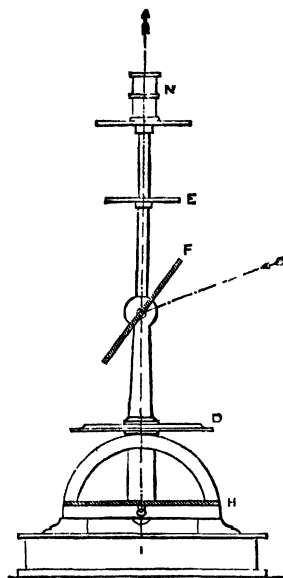
from the mirror ; and the polarised ray will have
passed, before it reaches the eye, twice through the
crystalline plate placed between the mirror and the
polariser.   The result is the same as if, in the ordinary
apparatus, the polarised ray had passed through a
plate of double the actual thickness.   If the plate
does not fill the entire field of view, two images of the
plate will be seen, the one larger, as viewed directly,

the other smaller, as viewed after reflexion from the horizontal mirror ; the first will show the effects due to the actual thickness of the crystal, the other that due to a plate of the same crystal, but of double the thickness.

It was mentioned in Chapter I. that light might from various circumstances become partially polarised ; and as this is not unfrequently the case in natural phenomena, such as the polarisation of the sky, or of the corona visible in a total solar eclipse, it becomes desirable to have some measure of the proportion of polarised light in a partially polarised beam. The following is a description of one such instrument, or polarimeter as it is called, devised by Prof. W. G. Adams.

This instrument consists of a box, about 4 inches square and 2 inches deep, inside which four parallel plates, very little smaller than the cross section of
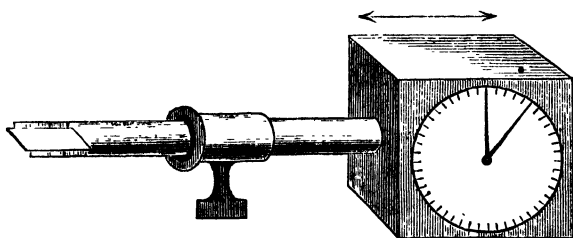

FIG. 13A.

the box, are attached to an axis passing through the middle points of the square faces of the box. On one of these faces is a circle graduated to degrees, and

an index attached to the axis and parallel to the
planes of the plates passes over this circle when the
axis is rotated, and shows the inclination of the plates
to the cross section of the instrument. Two circular
holes, each 1½ inch in diameter, are cut in the
centres of two opposite ends of the box ; into one
of these is fitted a double rotating right and left
handed quartz, with its plane of separation parallel
to the square faces of the box ; over this opening is
fitted one end of a tube about 10 inches long, and a
Nicol's prism is fitted to the other end.

The instrument is supported by means of a short
outer tube, to one end of which is attached a circle
graduated to degrees and which is supported steadily
on a light tripod stand. An index on the inner tube,
and revolving with it about its axis in front of the
graduated circle on the outer tube, shows the angle
through which the instrument is turned. When the
plane of separation of the biquartz is vertical, the
index on the tube should be at zero.

To use the polarimeter :

Place the Nicol's prism in the tube with its prin-
cipal section perpendicular to the plane of separation
of the biquartz ; place the glass plate so that their
planes are perpendicular to the axis of the tube, and
the index at zero. Light, entering at the opening at
one end of the box, passes perpendicularly through
the plates and the biquartz and Nicol's prism. If
the light is partially or wholly plane-polarised, turn
the tube and box until the plane of separation of the
biquartz coincides with the plane of polarisation of
the light, the index will then show the inclination of

the plane of polarisation to the vertical. Then turn the axis carrying the glass plates until the light passing through the Nicol's prism is found to be depolarised. The angle through which the plates are turned will give the proportion of plane-polarised light in the beam passing through the instrument, by means of a table of values depending on the angle of incidence of the light.

The mathematical theory of the instrument and a table of values for actual use are given by Prof. Adams in the *Philosophical Magazine* for March 1871.

CHAPTER III.

CHROMATIC POLARISATION—THE WAVE THEORY.

WE now proceed to the consideration of the colours produced by plates of crystal when submitted to the action of polarised light. A crystal very commonly used for this purpose is selenite or hydrated sulphate of lime, which is readily split and ground into flat plates of almost any required thickness. If such a plate be placed between the polariser and analyser when crossed, it will be found that there are two positions, at right angles to each other, in which, if the selenite be placed, the field will remain dark as before. The selenite is, in fact, a doubly refracting crystal, and although neither of the rays to which it gives rise follows the ordinary law, yet they may, for sake of distinction, be respectively called the ordinary and extraordinary. The positions in question are those in which the plane of vibration of the ordinary ray coincides with that of the polariser (or analyser), and that of the extraordinary ray with that of the analyser (or polariser). In every other position of the selenite, and notably when it has turned through 45° from either of the positions before mentioned, or neutral

positions as they may be called, light passes through, and the field becomes bright. If the thickness of the selenite be considerable, the field when bright will be colourless; but if it be inconsiderable, say not more than that of a thin visiting card, the field will be brilliantly coloured with tints depending upon the thickness of the plate.

Supposing however that, the selenite remaining fixed, the analyser be turned round, we shall find that, in the first place, the colour gradually fades as before; until, when the analyser has been turned through 45°, all trace of colour is lost. After this, colour again begins to appear; not, however, the original tint, but its complementary. And, in fact, there is no more sure way of producing colours complementary to one another than that here used. A general explanation of this change of colour is already furnished by our former experiments. Doubly refracting crystals generally, in the same way as Iceland spar, divide every ray, and consequently every beam of light, which passes through them, into two, so that of every object seen through them, or projected through them on to a screen, two images are produced. These two, being parts of one and the same beam of light, would, if recombined, reproduce the original beam; and the same is, of course, the case with the two images. This may be rendered visible by using the double-image prism as an analyser, and throwing both images on the screen together. It will then be seen that whatever changes the colours undergo when the prism is turned round, the overlapping part of the images is white; and their tints are therefore complementary.

If the distance of the prism be so adjusted that the images overlap, it will be found that, when both are visible, the part where they overlap is always white, whatever be the thickness of the plate used. And an instructive experiment may be made by interposing an opaque object in the path of the beam of light, so that its shadow may fall upon the part of the field common to the two images. The shadow will, of course, intercept the light forming each of the images, and will consequently appear double. Suppose that the two images are coloured red and green respectively ; then one of the shadows will be due to the shutting off of the red light, and the other to that of the green. But in the first case the space occupied by the shadow will be still illuminated by the green light, and in the second by the red. In other words, neither of the two shadows will be black; one will be green and the other red. If in any part of their extent the two shadows overlap, the part common to the two, being deprived of both red and green light, will be black.

But in order to explain how it comes to pass that colour is produced at all, as well as to find a more strict proof that the colours of the two images are complementary, we must have recourse to some considerations based upon the Wave Theory of Light. And, first, as to the mode in which waves may be produced.

Consider a row of balls lying originally in a horizontal straight line. Let the balls start one after another and vibrate at a uniform rate up and down. At each moment some will be at a higher, others at a

lower level, at regular intervals in a wave-like arrange-ment ; the higher forming the crests, the lower the hollows of the waves. The distance from crest to crest, or from hollow to hollow, is called the *wave length*. The distance from crest to hollow will con-sequently be half a wave length. This length will be uniform so long as the vibrations are executed at a uniform rate.

Each ball in turn will reach its highest point and form a crest ; so that the crests will appear to ad-vance from each ball to the next. In other words, the waves will advance horizontally, while the balls vibrate vertically.

If the row of balls were originally arranged in a wave form of the same amplitude, and caused to vibrate in the same way as before, those on the crests would vibrate wholly above, and those in the hollows wholly below the middle line. When the balls origi-nally on the crests rise to their highest positions, those in the hollows will fall to their lowest positions, and the height of the wave will consequently be doubled. When the balls originally at the crests fall, those in the hollows will rise, both to the middle line ; and the wave will consequently be annihilated. The first of these corresponds to a condition of things wherein the crests of the new wave motion coincide with those of the old, and the hollows with the hollows ; the second to that wherein the crests of the new coincide with the hollows of the old, and *vice versâ*.

Hence, when two sets of waves are coincident, the height of the wave or extent of vibration is doubled ; when one set is in advance of the other by half a wave

length, the motion is annihilated. The latter pheno-
menon is called *interference.* When one set of waves
is in advance of the other by any other fraction of a
wave length, the height of the wave, or extent of
vibration, is affected, but not wholly destroyed ; in
other words, partial interference takes place. The
distance whereby one set of waves is in advance of
another is called the *difference of phase.*

The Wave Theory of Light consists in explaining
optical phenomena by vibrations and waves of the
kind above described. And according to that theory
the direction in which the waves move is the direction
of propagation of the ray of light.

The intensity of light depends upon the extent of
the vibrations or the height of the waves ; the colour
upon the number of vibrations executed in a given
interval of time. And since throughout any uniform
medium the connexion of the parts and the rate of
propagation may be considered to be uniform, it
follows that the waves due to the slower vibrations
must be longer than those due to the more rapid.
Hence the colour of the light may be regarded as
depending upon the wave length.

The substance to the vibrations of which light is
supposed to be due is an elastic fluid or medium per-
vading all space, and even permeating the interior of
all bodies. A full statement of the reasons which
have led philosophers to make this hypothesis would
involve considerations derived from other sciences
beside optics, and would be out of place here. But
it may still be pointed out that one strong argument
is furnished by the fact of the transmission of light

from the sun and from the fixed stars through space, where no atmosphere or gases as known here can be conceived to exist. That the light so traversing interstellar space must be transmitted by a material substance is a fundamental proposition of mechanical philosophy ; and the hypothesis of the ether simply consists in attributing to the substance or medium the property of elasticity (a property possessed in a greater or less degree by all known bodies), without assuming anything else whatever as to its nature or relation to other substances.

In the illustrations of wave motions given above, the balls would represent successive portions or molecules of the ether ; and the means whereby the motion of one molecule is transmitted to its neighbour is their mutual attraction, or the elastic cohesion attributed to the whole medium in the hypothesis in question.

The difference between ordinary and polarised light has been explained above ; and the mechanical contrivances devised for representing wave motion always have reference only to polarised light. But as this is the subject with which we are here concerned, the limitation in question is not of consequence. A variety of instruments have been constructed for showing the motions of the particles of ether in the plane, circular, and elliptical vibrations of light ; and Sir Charles Wheatstone has invented a machine which represents to the eye in what manner plane waves in the same or in rectangular planes combine, so as to form every variety of plane, circular, or elliptical waves. It also exhibits the resultants of

two series of circular or elliptical waves, according to
their differences of phase; and it further shows that
when two circular waves, one right-handed and the
other left-handed, are combined, a series of plane
waves is formed, the azimuth of which changes with
the difference of phase.

In plane-polarised light, such as is produced by
tourmalin plates, by double refraction in Iceland spar,
&c., the vibrations are rectilinear, and are executed in
one and the same plane throughout the entire length
of the ray.   In circularly polarised light the vibrations
are all circular, and throughout the same ray the
motion is performed in the same direction.   In ellipti-
cally polarised light the vibrations are all elliptical, the
ellipses are all similarly placed, and the motion is in
the same direction for the entire ray.   These are the
only known forms of polarisation, and indeed they
are the only forms compatible with the usual, simplest
assumption respecting the elasticity of the ether.

These general considerations being premised, we
are in a position to trace the course and condition of a
ray of light issuing from the lamp or other source, and
traversing first the polarising Nicol's prism ; secondly,
the plate of doubly refracting crystal ; thirdly, the
analysing Nicol.

The vibrations of the ray on leaving the polariser
are all restricted to a single plane.   On entering the
plate of doubly refracting crystal, every ray is divided
into two whose vibrations take place in planes perpen-
dicular to one another.   The angular position of these
planes about the axis of the beam of light is dependent
upon the angular position of the crystal plate about its

centre. The two sets of rays traverse the crystal with
different velocities, and therefore emerge with a differ-
ence of phase. The amount of this difference is pro-
portional to the thickness of the plate. On entering
the analyser the vibrations of each pair of rays are
resolved into one plane ; and are then in a condition to
exhibit the phenomena of interference. If the plane
of vibration of the analyser be parallel to one of those
of the plate, that ray will be transmitted without
change ; the other will be suppressed. In any other
position of the analyser those monochromatic rays
(spectral components of white light) whose difference
of phase most nearly approaches to half a wave length
will be most nearly suppressed ; and those in which it
approaches most nearly to a whole wave length will be
most completely transmitted. The amount of light
suppressed increases very rapidly in the neighbourhood
of the ray whose difference of phase is exactly a half-
wave length ; so that with plates of moderate thickness
a single colour only may, speaking roughly, be con-
sidered to be suppressed. This being so, the beam
emergent from the analyser will be deprived of that
colour, and will in fact consist of an assemblage of all
others ; or, in other words, will be of a tint complemen-
tary to that which has been extinguished.

Next, as regards the colours of the two images --
that is, the two which are formed either simultaneously
by a double-image prism, or successively by a Nicol
in two positions at right angles to one another. In the
first place, it is to be remembered that the two sets of
vibrations into which the selenite has divided the
polarised ray are at right angles to one another ;

secondly, that one set is retarded behind the other through a certain absolute distance, which is the same for every ray, and consequently through a distance which is a different fraction of the wave length for each colour ; thirdly, that these two are re-combined or " resolved " in a single direction in each image by the analyser.

This being so, bend two wires in the following form :—
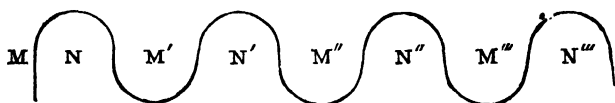
and place them at right angles to one another about their middle line M N M' N' . ., so that the points M of the two wires coincide, and likewise N, and so on. This will represent the condition of the vibrations as they emerge from the selenite, when the plate is of such a thickness as to cause a retardation equal to one or to any whole number of wave lengths. Turn the wires about their middle lines M N M' N' until they meet half-way, *i.e.* in a position inclined at 45° to their original directions ; this will represent the vibrations as resolved by the analyser in one image. Turn the wires about their middle line as before, but in reversed directions, until they meet in a position at right angles to the former ; this will represent the vibrations as resolved by the analyser in the other image. On looking at the wires when so brought together, it will be found that in one case the crests fall upon the crests and the hollows upon the hollows,

so that the vibrations combine to increase the intensity of the light.   In the other case the crests fall upon the hollows and the hollows upon the crests, so that the vibrations interfere and completely neutralise one another.

The same principle would obtain if we shifted one wire along the middle line so that the points M of the two wires no longer exactly coincide.   This would represent the condition of the vibrations as they emerge from the selenite when the plate is of such a thickness as to cause a retardation of a fraction of a wave length equal to the amount of shift.   And on turning the wires as before, we should find that in one image the waves partially combine, and that in the other they partially interfere.   The shifting of the wires would represent either the effect of plates of different thicknesses upon waves of the same length, *i.e.* rays of the same colour ; or that of a single plate on waves of different lengths, *i.e.* on rays of different colours.   From these considerations we may conclude that the rays which are brightest in one image are least bright in the other ; or, in other words, that the colours of the two images are complementary.

It has been remarked that the colour produced by a plate of selenite depends upon the thickness of the plate.   In fact, the retardation increases with the thickness, and consequently, if, for a given thickness, it amounts to a half-wave length of the shortest (say violet) waves, for a greater thickness it will amount to a half of some longer (say green) waves, and so on. And if, instead of a series of plates of different thicknesses, we use a wedge-shaped plate, the entire series

of phenomena due to gradually increasing retardation will be produced. This is easily seen to consist of a series of tints due to the successive extinction of each of the rays, commencing with the violet and ending with the red. And the tints will consequently have for prevailing hues the colours of the spectrum in the reverse order. This series of colours will be followed by an almost colourless interval, for which the retardation is intermediate between a half-red wave length and three half-violet wave lengths. Beyond this again the series of colours will recur; and the same succession will be repeated as the wedge increases in thickness. It will, however, be observed that the colours appear fainter each time that they recur, so that when the thickness reaches a certain amount (dependent upon the nature and retarding power of the crystal) all trace of colour is lost.

It is not difficult to account for this gradual diminution in the intensity of the colours, if, by means of a diagram, we examine the mode in which the waves of various lengths interfere with one another; but spectrum analysis furnishes an explanation which is perhaps more easy of general apprehension. If the light emerging from the analyser be examined by a spectroscope, it will be found, in the case of a plate giving the most vivid colour, that the spectrum presents a single dark band indicating the colour which has been extinguished. On using thicker and thicker plates, the band will be found to occupy positions nearer and nearer to the red end of the spectrum, until the band finally disappears in the darkness

beyond the least refrangible rays that are visible to the eye. If the thickness of the plate be further increased, two bands will be seen instead of one ; with a still greater thickness there will be three bands, and so on indefinitely. It follows, therefore, that the thicker the plate the greater is the number of colours suppressed ; and consequently the more nearly will the tint resulting from a combination of those colours approach to white. But the resultant of the colours suppressed in one image is to the eye the same as the tint of the other image ; because they are each complementary to the tint of the first image. Hence the thicker the plate, the more nearly will the tint of each image approach to white.

Further, whatever be the thickness of the plate, if the analyser be turned round, the bands will gradually lose their characteristic darkness, until, when the angle of rotation has reached 45°, the bands will have disappeared altogether. The spectrum is then continuous, and when recompounded will give white light. This corresponds to the fact noticed before, that when the analyser is turned round, the colour given by a selenite plate fades, and finally disappears when the angle of rotation amounts to 45°. If the rotation be continued, the bands reappear, not, however, in their original position, but in intermediate parts of the spectrum. The parts of the spectrum occupied by the bands in the two rectangular positions of the analyser are complementary to one another ; as will be more fully explained hereafter.

These results undergo a curious modification if

instead of a flat plate of crystal we use a wedge,
thicker, say, at its lower than at its upper edge; in
which case it is not difficult to predict the result.   If
at the bottom the thickness be such as to produce ‚'
band in a given position, say the blue, then at a point
higher up the thickness being greater would produce
a band nearer to the red, say in the green ; at a point
still higher the thickness will be such as to produce
a band still nearer the red, say in the yellow, and so
on.   Now as the thickness of the wedge increases
gradually, so will the shifting of the band from the
blue towards the red also take place gradually; that
is to say, the band or bands will, instead of appearing
as vertical straight lines, present a slope or curvature
from blue towards red, in the direction of the thicker
side of the wedge.

The following experiment will be found very in-
structive. Take two wedges of selenite or other crystal,
and having crossed the polariser and analyser, place
the two wedges side by side in the field of view so as
to compare the tints produced by the two. Then place
one over the other, first with the thick end of the one
over that of the other, next with the thick end of the
one over the thin end of the other.   If the two plates
are exactly similar, the combination in the first instance
will be equivalent to a single wedge whose refracting
angle is double that of a single wedge ; and the num-
ber of bands produced will consequently be doubled.
In the second combination the angles of the wedges
will compensate one another, and the result will be
equivalent to a uniform plate whose thickness is equal

to the sum of the mean thicknesses of the wedges. The field will then be coloured with a uniform tint, viz. that due to a plate of the thickness in question.

If we take a concave crystal, such as without the slit or dispersive prism would give concentric rings of colour, the part to which the slit is opposite will be nearly like a double wedge thinnest at the centre and increasing in thickness towards both top and bottom. The bands ought therefore to slope towards the red from the centre in the direction of both the upper and the lower edges. They will, however, sweep *round more rapidly with the concave plate than with* the wedge because the thickness increases more rapidly towards the circumference than near the centre.

If we take two wedges cut with the axis of one parallel, and that of the other perpendicular, to the thin edge (Babinet's compensator), the two pieces can be arranged in the following ways :

(1.) With their edges coincident, and their axes at right angles, in which position each neutralises the effect of the other ; an unbroken spectrum will be the result.

(2.) With their edges at right angles, and their axes parallel, and both placed at 45° to the plane of polarisation. In this case we have two positions : (*a*) when the line of increasing thickness of the compound piece is parallel to the slit, the bands are parallel to the colours of the spectrum ; (*b*) when the line is perpendicular to the slit, we have ·diagonal bands. In both cases we have an increase in the

number of the bands when the compound piece is moved so as to present a greater thickness to the slit.

(3.) With their edges opposed, and their axes at right angles. Here also we have two cases : (*a*) when the axes and edges are placed at 45° to the plane of polarisation, we have a fan-shaped system of bands, opening out towards the red, because the centre is the thickest part of the compound piece ; (*b*) when the piece is turned round so as to bring the edges *and axes parallel or perpendicular to the plane of* polarisation, the bands separate and become gradually wider, until all trace of them is lost in a bright or dark field.

(4.) With their edges, and consequently also their axes, at 45° to one another. The bands due to the two plates are then both diagonal, the one dexter, the other sinister, and when the compound piece is so placed that both sets of bands are actually formed, the two systems cross one another and form a diaper pattern, bright on a dark ground, or dark on a bright one, according to the position of the piece and the analyser.

By making use of the principle that the colour produced depends upon the thickness of the plate, selenites have been cut of suitable shapes and thicknesses, so as to produce coloured images of stars, flowers, butterflies, and other objects.

The phenomena exhibited by selenite are also produced by other crystals, but the facility with which plates of the former substance can be obtained causes them to be generally used in preference to others.

There is, however, a peculiar class of crystals, of which quartz, or rock crystal, is the most notable, and which gives rise to effects different from those hitherto described. These will form the subject of the next chapter.

Before quitting this part of the subject it will be worth while to notice a form of analyser different from those hitherto described. This consists of a double-image prism cut so as to show one image in the centre of the field and the other excentric, and mounted so as to admit of a rapid motion of rotation. If the speed attains eight or ten revolutions per second, the moving image will remain permanently on the retina during an entire revolution, and all the phenomena which are usually seen in succession will appear simultaneously displayed in a circle or ring. The central image will consist of a superposition of the images due to the successive azimuths of the analyser, and will consequently appear unchanged in brightness or in colour during the working of the instrument. In particular, if the polariser and analyser be used without any interposed plate, the excentric image will be brightest at two positions opposite to one another, say at 0° and 180°, and dark at 90° and 270°. A rapid revolution of the analyser will therefore (as in the case of the sphere of spar) give the appearance of a ring brightest in the two positions first mentioned, and fading into darkness at the two other positions.

If a plate of selenite be interposed the ring will appear coloured in opposite quadrants with the same

tint, in the intermediate quadrants with the complementary tint. In the intervening parts the tints will fade into one another. If a plate of quartz perpendicular to the axis be used instead of the selenite, the entire series of tints will be seen displayed twice over the ring. Many other experiments with this revolving analyser will suggest themselves to the reader.

CHAPTER IV.

CIRCULAR POLARISATION.

IF a ray of light pass through a plate of quartz which has been cut perpendicularly to the axis, or line parallel to the length of the crystal (hexagonal prism), it is as usual divided into two moving with different velocities ; but the vibrations in each ray, instead of being rectilinear and at right angles to one another, are circular and in opposite directions. That is to say, if the motion of vibration in one ray is directed like the hands of a clock, that in the other is directed in the opposite sense ; and the light in each ray is then said to be circularly polarised. The motion of a series of particles of ether, which when at rest lie in a straight line, is circular, and, as in plane polarisation, successive ; and consequently at any instant during the motion such a series of particles will be arranged in a helix or corkscrew curve. The sweep of the helix will follow the same direction as that of the circular motion ; and, on that account, a circularly polarised ray is spoken of as right-handed or left-handed, according to the direction of motion. A right-handed ray is one in which, to a person looking

in the direction in which the light is moving, the plane of vibration appears turned in the same sense as the hands of a watch. Or, what is the same *thing, to a person meeting the ray, it appears turned in the opposite sense, viz. that in which angles* when measured geometrically are usually reckoned as positive.

The question, however, which mainly concerns us is the condition of the vibrations after emerging from the plate of quartz and before entering the analyser. In the passage of the ray through the place the ether is subjected to a double circular motion, one right-handed, the other left-handed. But, as one of these motions is transmitted with greater velocity than the other, it follows that at any given point and at the same instant of time one of the revolutions will, in general, be more nearly completed than the other ; or, to use an expression adopted in plane polarisation, there will be a difference of phase. The motions may be represented by two clock hands moving at the same rate in opposite directions, and the difference of phase by the angle between them when one of them is in the position from which angles are reckoned. And if we consider a particle of the ether (say) at the extremity of the clock-hands when they have met, it will be solicited when the hands are coincident by forces producing two opposite circular motions. Now, whatever may have been the forces or structural character within the crystal whereby this double circular motion is perpetuated, it is clear that when the ray emerges into air the particle of ether immediately contiguous to the surface

of the crystal will be acted on by two sets of forces, one whereby it would be caused to follow the right-handed and the other the left-handed rotation. Each of these may, as is well known, be represented by a pair of forces, one directed towards the centre of the circle, the other in the direction of the motion and at right angles to the first, or, to use geometrical language, one along the radius and towards the centre, the other along the tangent and in the direction of the motion. The two forces acting along the tangent, being in opposite directions, will neutralise one another, and the resultant of the whole will, therefore, be a force in the direction of the centre. The particle in question, and consequently all those which, following in succession, serve to compose the entire ray until it enters the analyser, will vibrate in the direction of the diameter drawn through the point under consideration ; or, to express it otherwise, the ray will be plane-polarised, and the plane of vibration will be inclined to the plane from which angles are measured by an angle equal to half the difference of phase on emergence due to the thickness of the crystal. . The retardation being very nearly the same, absolute quantity for all rays will, as in the case of plane polarisation, be a different fraction of the wave length for rays of different colours, and will be greater for the shorter waves than for the longer. Hence the planes of vibration of the different coloured rays, after emerging from the quartz, will be differently inclined. Each ray will therefore enter the analyser in a condition of plane polarisation ; and if the analyser be turned round, it will cross the vibrations

of the various coloured rays in succession, and extinguish each of them in turn. Each of the images will consequently exhibit a gradual change of colour while the analyser is being turned; and the tints will be, as explained before, complementary to those which are successively extinguished. For a given plate of quartz the order of the tints will be reversed when the direction of rotation of the analyser is reversed. But it should be here explained that there are two kinds of quartz, one called right-handed and the other left; and that, for a given direction of rotation of the analyser, these cause the colours to follow one another in opposite orders. A similar effect is produced by turning the polariser round in the opposite direction.

The angle of rotation of the plane of vibration for any particular colour varies, as stated above, with the thickness of the plate; while for a given thickness it increases nearly as the square (product of the quantity into itself) of the wave length decreases. In mathematical language, it varies approximately inversely as the square of the wave length. If this law were accurately true, the product of the angles of rotation into the square of the corresponding wave lengths (λ) would be the same for all rays. The following are some measurements made by Brock, with a quartz plate one millimetre thick, which show that the law may be considered as true for a first approximation.

Rays.	Rotations.	Rotations x λ^2.
B	15° 18′	7,238
C	17° 15′	7,429

Rays.	Rotations.	Rotations × λ^2.
D	21° 40′	7,511
E	27° 28′	7,596
F	32° 30′	7,622
G	42° 12′	7,842

If the colours exhibited by a plate of quartz when submitted to polarised light be examined by a spectroscope, in the way described when we were speaking of selenite, the spectrum will be found to be traversed by one or more dark bands, whose position and number depend upon the thickness of the plate. But there will be this difference between plane and circular polarised light, that if the analyser be turned round the bands will never disappear, but will be seen to move along the spectrum in one direction or the other, according as the plate of quartz be right-handed or left-handed, and according to the direction in which the analyser is turned. This is, in fact, identical with the statement made before, that the analyser in its different positions successively crosses the plane of vibration of each ray in turn, and extinguishes it.

This being so, it is clear that a change of colour exhibited by a quartz plate, when submitted to plane-polarised light and examined with an analyser, forms a test of a change in the plane of original polarisation. And if the plate be composed of two parts, one of right-handed the other of left-handed quartz, placed side by side, any change in the plane of polarisation will affect the two parts in opposite ways. In one part the colours will change from red to violet ; in the other from violet to red. At two positions of the polariser, or analyser, the colours must be identical.

With plates, as usually cut, one of these identities will
be in the yellow, the other at the abrupt passage from
violet to red, or *vice versâ*. In the latter case the
field appears of a neutral tint—*teinte sensible* or *teinte
de passage*, as the French call it—and the slightest
change in the plane of polarisation exhibits a marked
distinction of colour, one part verging rapidly to red,
the other to violet. This arrangement is called a
biquartz, and affords a very delicate test for deter-
mining the position, or change of position, of the
plane of polarisation, especially in cases where feeble-
ness of light or other circumstances interfere with the
employment of prismatic analysis.

If the thickness of the plate be such that the dif-
ference of rotation of the planes of vibration of the
rays corresponding to the two ends of the visible
spectrum (or, as it is sometimes termed, the " arc of
dispersion ") be less than 180°, there will be one dark
band in the spectrum ; because there can then be only
one plane of vibration at a time at right angles to that of
the analyser. If the arc of dispersion is greater than
180° and less than 360°, there will be two bands.
And so on for every 180° of dispersion.

This mode of examination by means of prismatic
analysis is the most accurate yet devised for mea-
suring the angle of rotation produced by circular
polarisation ; especially if solar light be employed,
and the fixed lines used to form a scale of measure-
ment.

The property of circular polarisation is, however,
not confined to quartz.

The following crystals are known as having the property of rotatory polarisation.

In the cubic system—

Sodium chlorate.
„ bromate.
„ iodate.

In the tetragonal system—

Strychnine sulphate crystallised with 13 equivalents of water.
Æthylodiamine sulphate.

In the hexagonal system—

Quartz.
Cinnabar.
Sodium periodate ($NaIO_4 + 2H_2O$).
Lead hyposulphate ($PbS_2O_6 + 4H_2O$).
Benzil.
(Possibly also some other hyposulphates.)

Sir John Herschel was the first to observe a remarkable connection between the external form and the direction of rotation produced by different specimens of quartz. The normal form of the crystal is that of a hexagonal prism, terminated at each end by hexahedral pyramids. The angles at the ends of the edges of the prism are sometimes cut off by little facets, which in some cases are inclined to the right, in others to the left, when the prism is held vertically. But when the upper facet is inclined to

the right, the lower facet is inclined to the left, and *vice versâ*; and the direction of the rotation produced by the crystal follows the inclination of the upper facet.

In the language of crystallography these facets are termed hemihedral planes. And from the fact that two planes on the same edge of the prism are always inclined in opposite directions, it follows that a right-handed and a left-handed specimen cannot be geometrically superposed; one is in fact the reflected image of the other. The formal relation between the two kinds has on this account been termed hemihedral hemi-symmetry.

Quartz and sodium periodate and lead hyposulphate are the only crystals in which right and left handed rotatory polarisation has been identified with hemihedral hemi-symmetry.

Beside crystals, other substances possess the property of turning the plane of polarisation in the manner above described. The following list is given by Verdet. The angles have reference to the red rays given by a plate of glass coloured with oxide of copper, and are affected with the sign + in the case of right-handed, and with − in the case of left-handed rotation. The length of the column of the solution is in every case one decimetre.

Essence of turpentine .	.	.	−29° ·6
,,	lemon	.	.	. +55° ·3
,,	bergamot .	.	.· +19° ·08
,,	bigarade .	.	. +78° ·94

Essence of aniseed	.	.	.	− 0° ·70
„	fennel	.	.	.	+ 13° ·16
„	caraway	.	.	.	+ 65° ·79
„	lavender	.	.	.	+ 2° ·02
„	peppermint	.	.	− 16° ·14
„	rosemary	.	.	.	+ 2° ·29
„	marjoram	.	.	.	+ 11° ·84
„	sassafras	.	.	.	+ 3° ·29
Solution of sugar 50 per cent.	.	+ 33° ·64
„	quinine 6 per cent. in ⎱ alcohol . . ⎰	− 30°

It will be noticed that the rotatory power of all these substances is much less than that of quartz.

A mixture of liquids, one or both of which is active, generally exhibits a rotatory action represented by the sum or difference of their separate powers (a neutral liquid being considered to have a power represented by 0) ; but this law is true only when no chemical action takes place between the elements of the mixture. Saccharine solutions vary not only in the amount but also in the character of their power of rotation ; thus cane sugar is right-handed, but grape sugar left-handed.

The property in question has been turned to practical use by employing the rotatory power of a saccharine solution as a measure of the strength of the solution. For this purpose a tube containing the solution to be examined is placed between two Nicol's prisms. The simple fact of circular polarisation is proved by the feeble exhibition of the

phenomena shown by a plate of quartz cut perpendicularly to the axis. But for accurate measurement various expedients have been adopted. If a biquartz be inserted behind the analyser (the end of the apparatus next the eye being considered the front), then for a certain position of the analyser the two halves will appear of the same colour. When the tube for examination is inserted, the similarity of colour will be disturbed; and the angle through which, right or left, the analyser must be turned in order to restore it will be a measure of the rotatory power of the fluid.

Another method is as follows :—Use a single quartz instead of a biquartz ; in front of it place a pair of quartz wedges with the thin end of one opposite the thick end of the other ; the outer surfaces having been cut perpendicularly to the axis. If the plate be right-handed, the wedges must be left-handed, and *vice versâ*. The wedges are made to slide one over another so as together to form a plate of any required thickness, and a scale connected with the sliding gear registers the thickness of the plate produced. When the tube is removed, the wedges are adjusted so as to compensate the quartz plate, and their position is considered as the zero point of the scale. When the tube is replaced, the wedges are again adjusted so as to compensate the action of the fluid in the tube, and the difference of the readings gives the thickness of quartz necessary for the compensation. The rotatory effect of a given thickness of quartz being supposed known,

we know at once the effect of a thickness of the
fluid under examination equal to the length of the
tube.

Returning to the subject of quartz ; if a Nicol's
prism be used as a polariser and a sphere of Iceland
spar as an analyser, then, when the sphere is so
placed that its axis coincides with that of the beam
of incident light, two images are formed on the
screen. If the aperture be a circular ring, the two
images will be concentric circles ; the ordinary image
being the larger, and the extraordinary the smaller.
The light in both images is polarised, but the polarisa-
tion in one is radial, in the other tangential. Hence
each ring will be broken by two dark intervals oppo-
site to one another, viz. at points where the vibrations
would be perpendicular to those transmitted by the
polariser ; and as the vibrations at each point of one
ring are perpendicular to those at the correspond-
ing point of the other, the dark intervals in one
ring are in quadrants at right angles to those in the
other.

If monochromatic light be used, the same effect
is seen as with white light ; but if a plate of quartz
cut perpendicularly to the axis be inserted between
the Nicol and the sphere, the dark interval will be
shifted to the right or to the left according as a right-
handed or a left-handed quartz is used. The amount
of displacement will depend, for light of a given
colour, on the thickness of the quartz ; and, for a
given thickness of quartz, on the colour, or, more
strictly speaking, on the wave length. In this way

the rotation of the plane of polarisation by the quartz plate is brought visibly before the eye.

In this experiment, however, the various stages of rotation for the rays of various colours are brought successively into view; but by the following arrangement they may be all exhibited simultaneously. If, while white light is used, and spectra of the ring images be formed, then, on re-introducing the quartz plate, the spectrum of one ring (the ordinary, or the extraordinary, according to the position of the polariser) will be seen to be crossed by a spiral band of darkness, sweeping obliquely from red to blue, or *vice versâ*, according as the quartz is right-handed or left-handed. This shows that the various prismatic colours in continuous succession are extinguished at various points of the ring; or, in other words, that the plane of polarisation varies continuously in angular position with the wave length. This fact explains the effect seen when the image is formed on the screen without dispersion; viz. the complete image consists of two concentric rings coloured with the residual tints (arising from the extinction of the prismatic colours in succession). These tints run through their cycle once in each semi-circumference; and the tints in each semicircle of one image are complementary to those in the corresponding semi-circle of the other. If a biquartz be used, and the line of division between the right-handed and left-handed portions pass through the junction of the red and violet parts of either of the rings, the order of colours in the two semicircles will be reverse to one another

If with the same arrangement as last described the sphere be turned round through an axis parallel to the line of division of the biquartz, the following results may be observed: first, each of the two images will be tinted by the biquartz in two compartments, the tints of which depend upon the position of the Nicol; secondly, the two images will, as usual, present tints complementary to one another; thirdly, the parts where the complementary tints overlap will, as usual, appear white; fourthly, the parts where the same tints overlap will be more brightly illuminated than the parts which do not so overlap. The interest of the last feature consists in showing, as first noticed by Helmholtz, that the low-tint colours, russet, brown, drab, &c., are really subjective effects due to red, orange, yellow, &c., when feebly illuminated in comparison with some other brighter part of the field.

The effect of right and left handed quartz in combination is ordinarily shown, and turned to useful account, by a combination of plates, as in Savart's bands, or of wedges, as in Babinet's compensator; but it may also be well illustrated by the following arrangement. A solid cone, having a very obtuse vertical angle, and its axis parallel to that of the crystal, is cut from a right-handed quartz; a second cone, in all other respects similar to the first excepting that it is hollow, is cut from a left-handed quartz; and the two are cemented, one inside the other, by balsam, so as to form a compound plate. When the polariser and analyser are crossed, the following

phenomena are observed : The coloured bands are arranged in circles about the centre. Midway between the centre and circumference, where the thicknesses of right-hand and left-hand quartz are equal, a black ring is seen. Inside this the colours are arranged in the order due to right-handed quartz ; outside it they are arranged in the order due to left-handed quartz. On turning the analyser round, the middle ring changes from black to white, and the other rings, within and without the middle ring respectively, change their colours in opposite orders. But owing to the fact that the colours are originally ranged in opposite orders, the last-mentioned change gives rise to an optical illusion, in virtue of which the coloured rings seem to flow inwards or outwards throughout, according to the direction in which the analyser is turned.

Circular polarisation may, however, be also pro-duced by other means than circularly polarising media properly so called. It may be brought about by total reflexion, and by transmission through ordinary·doubly refracting plates of suitable thick-ness.

Of these various methods, it will, perhaps, be best to begin with the last. And, in order the better to understand the process, we must consider briefly the result of compounding two rectilinear vibrations under different circumstances.

Suppose a particle of ether to be disturbed from its point of rest O in a direction O A. The attraction of the particles in its neighbourhood would tend to

draw it back to O; and let O A be the extreme dis-
tance to which under these attractions it would move.
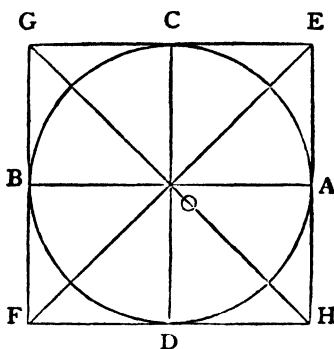
Having reached A, it would return to O; and passing
through O with a velocity equal to that with which it
started under the disturbing force, it would move to
a point B equidistant from O with A, but in the oppo-
site direction. And if, as is generally supposed, the
ether is perfectly elastic, or that there are no internal
frictions or other conditions whereby the energy of
motion is converted into other forms of energy, the
oscillations or vibrations of the particle between the
points A and B will continue indefinitely. Now, sup-
pose that while these vibrations are going on, a
second disturbing impulse equal in intensity, but in a
direction at right angles to the first, be communicated
to the particle; it is clear that the effect on the motion
of the particle will be different according as it takes
place at the point of greatest velocity O, or at one of
no velocity, A or B, or at some intermediate point.

Our object is to consider the effects under these various circumstances.

A complete vibration consists in the motion from O to A, thence to B, and finally back to O ; so that if O be the starting point, the passage through A will be removed one-fourth, the passage through O from A towards B will be one-half, the passage through B will be three-fourths, and the passage through O from B to A a complete vibration from the commencement. This being so, suppose that the second impulse be communicated while the particle is at O on its way towards A, then the impulses may be considered as simultaneous, and the vibrations to which they give rise will commence together, and the waves of which they form part will be coincident. If the second impulse take place when the particle is at A, the two sets of vibrations or waves to which they belong will have a difference of phase (*i.e.* the first will be in advance of the second) equal to one-fourth of a vibration or onefourth of a wave length. If the second impulse take place when the particle is at O on its way to B, the difference of phase will be half ; if when it is at B, the difference will be three-fourths of a wave length.

The particle, being at O, and subject to two simultaneous impulses of equal strength, one in the direction of A, the other in that of C, must move as much in the direction of C as in that of A ; that is, it must move in a straight line equally inclined to both, namely, O E in the same figure. And inasmuch as the two impulses in no way impede one another, the particle will move in each direction as far as it would have done if the other had not taken place. In other

words, if we draw a square about O, with its sides at
a distance equal to O A or O B, the extent of the vibra-
tion will be represented by O E, where E is a corner of
the square. The complete vibration will then be re-
presented by the diagonal E F, in the same way as it
was by the line A B in the first instance. If the
impulse had been communicated at the instant of
passage through O on the way to B, it is clear that a
similar train of reasoning would have shown that the
vibration would have been in the other diagonal G H.
We conclude, therefore, that if two sets of rectilinear
vibrations, or plane waves, at right angles to one
another, combine, then when they are coincident they
will produce a rectilinear vibration, or wave, whose
plane is equally inclined to the two, and lying in the
direction towards which the motions are simul-
taneously directed. In the figure this is represented
by the dexter diagonal. When the two sets of waves
have a difference of phase equal to half a wave
length, their combination gives rise to a wave repre-
sented in the figure by the sinister diagonal.

Suppose now that the second impulse is commu-
nicated at the instant when the particle is at A ; in
other words, that the two sets of waves have a dif-
ference of phase equal to one-fourth of a wave length.
At that instant the particle will have no velocity in
the direction of A B (for convenience, say eastwards),
and will consequently begin to move in the direction
of the second impulse, say northwards. But as time
goes on, the particle will have an increasing velocity
westwards and a diminishing velocity northwards ; it
will therefore move in a curve which grádually and

uniformly bends, until, when it has reached its greatest distance northwards, it will be moving wholly west-wards. And as the motion not only will be the same in each quadrant, but would be the same even if the directions of the impulses were reversed, it is clear that the curvature of the path will be the same throughout ; that is to say, if two sets of waves of the same magnitude in planes perpendicular to one another, and with a difference of phase equal to one-fourth of a wave length combine, they will produce a wave with circular vibrations.

If the second impulse be given when the particle arrives at B—that is, if the waves have a difference of phase equal to three-fourths of a wave length—similar considerations will show that the motion will be cir-cular, but in the opposite direction.

Suppose, therefore, that we allow plane-polarised light to fall upon a plate of doubly refracting crystal cut perpendicularly to the axis in the case of a uni-axal crystal, or in the case of a bi-axal to the plane containing the two axes—say a plate of mica, which splits easily in that direction—then the vibrations will, as before explained, be resolved in two directions, at right angles to one another. And further, if the original directions of vibration be equally inclined to the new directions, *i.e.* if it be inclined at 45° to them, the amount or extent of vibration resolved in each direction will be equal. Further, if the thickness of the plate be such as to produce retardation or differ-ence of phase equal to a quarter of a wave, or an odd number of quarter-wave lengths, for the particular ray under consideration, then the two sets of vibrations on

emerging from the mica plate will recombine, and, in accordance with the reasoning given above, they will form a circular vibration, left-handed or right-handed according as the retardation amounts to an integral number of three-quarter wave lengths or not.

It thus appears that a plate of mica, or "quarter undulation plate," which retards one of the sets of waves into which it divides an incident set by an odd multiple of quarter-wave lengths, affords a means of producing circular from plane polarisation. It remains to be shown that, with the same plate in different positions, right or left-handed circular polarisation may be produced at pleasure. Suppose that the original vibrations are in the direction E F in the foregoing figure ; the mica plate will resolve them into the two directions A B, C D, one of the rays—say the first—will be transmitted with greater velocity than the other, and the vibrations along C D will be one-fourth of a wave length behind those along A B. This will correspond to the case discussed above, and will give rise to a circular vibration in a direction opposite to that of the hands of a clock. Suppose, however, that the plate be turned round through a right angle, so that the vibrations which are transmitted with greater velocity are placed parallel to C D, and those which are transmitted with lesser along A B. The ray whose vibrations are along A B will then be a quarter-wave length in advance, or, what comes to the same thing, they are three-quarters of a wave length in rear of the others ; and this condition of things produces, as explained before, a circular vibration in a direction the reverse of the former. It

thus appears that the plate placed in one direction will convert plane into right-handed circular polarisation; and if turned round through a right angle from that position will convert plane into left-handed circular polarisation. A like change from right-handed to left-handed circular polarisation, or *vice versâ*, may obviously be effected by turning the original plane of polarisation through a right angle ; so that it shall lie between lines of concurrent instead of between lines of discordant motion.

It must be borne in mind that the above remarks refer to light of one colour only, because the retardation due to the thickness of the plate will amount exactly to a half-wave of one definite length only. But if we take a thickness calculated to cause such a retardation in a yellow ray, at or near one of the sodium lines, the plate will be found suitable for general approximate use with all luminous rays. This observation, of course, will apply, *mutatis mutandis*, to all experiments with quarter-undulation plates.

Beside the rotation of the plane of polarisation due to the action of certain natural substances, described in a former part of the present chapter, Faraday discovered as long ago as 1846 that, if a ray of plane polarised light be passed through a piece of " heavy glass " (as it is called) in a line joining the poles of a magnet, the plane of polarisation is turned through an angle by the action of the magnet. The direction of rotation was found to depend upon the polarity, and the amount upon the strength of the magnet. But between the rotation produced by

natural substances and that produced by a magnet
there is this essential difference : if the light which
has passed once through the glass be reflected back
so as to pass through it a second time, the amount
of rotation is doubled ; whereas if the same operation
be performed on a ray which has passed through a
substance of natural rotation, the rotation is annulled.
In the first case the rotation is continuous and inde-
pendent of the direction of the passage of the light ;
in the second it is determinate like a screw-thread
cut in the substance itself, so that the direction is
reversed with that of the ray of light.   The action
thus produced by a magnetic field may also be pro-
duced by a helix of wire carrying a current, *i.e.* by a
solennoid, which, as is well known, may be used in a
great many experiments to replace a magnet.   The
subject thus opened out by Faraday was subsequently
developed experimentally by Verdet, by H. Bec-
querel, by J. E. H. Gordon, and theoretically by
Clerk Maxwell.

In 1875 and 1877 respectively, Dr. Kerr, of
Glasgow, announced two remarkable discoveries :
(1) that when glass and certain other dielectrics are
subjected to an intense electric strain, they acquire
the power of double refraction ; and (2) that when
plane polarised light is reflected regularly from either
pole of an electro-magnet of iron, the plane of polari-
sation is turned through an angle in a direction
contrary to the nominal direction of the magnetising
current.

It would exceed the limits permissible in this

little work to enter further into the details of these
very interesting subjects; but the reader will find a
very full and clear statement of all that has been
done in them up to date in J. E. H. Gordon's
" Physical Treatise on Electricity and Magnetism "
(Sampson, Low & Co.).

CHAPTER V.

CIRCULAR POLARISATION BY REFLEXION.

THE conversion of plane into circularly polarised light may also be effected by total reflexion.   If plane-polarised light traversing glass be incident upon the inner side of the limiting surface at any angle at which total reflexion takes place, it may be considered as resolved into two plane-polarised rays, the vibrations of one being parallel and those of the other perpendicular to the plane of reflexion ; and there is reason to believe that in every such case a difference of phase is brought about which for a particular angle in each substance (in St. Gobain glass it is 54° 30′) it has a maximum value somewhat in excess of one-eighth of a wave length.   There are two angles, one on each side of that giving the maximum, for which the difference of phase is one-eighth of a wave length.   And if the original plane of vibration be inclined at an angle of 45° to that of reflexion, the amplitudes of the two vibrations, into which the reflected vibrations are supposed to be resolved, will be equal.   A full discussion of the mechanical causes which may be considered to effect this difference of phase would carry us deeper into the more

difficult parts of the Wave Theory than would be suit-
able in this place.   But if we accept the fact that the
above-mentioned effects result, when polarised light
(whose plane of vibration is inclined at 45° to that of
reflexion, so as to ensure an equality of amplitudes in
the components) is reflected at a proper angle, then
the following construction will be readily understood.
Take a rhomb of glass, *a, b, c, d,* Fig. 14, whose
acute angles are 54° 30′ ; a ray incident perpen-
dicularly to either end will undergo two total internal
reflexions at the sides—say at *p* and *s*—and will emerge
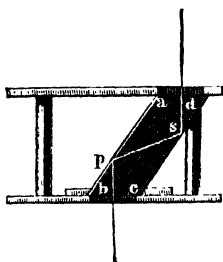
FIG. 14.

perpendicularly to the other end.   These two reflex-
ions will together produce a retardation, as described
above, of one-fourth of a wave length.   And if the
ray be originally polarised, and its plane of vibration
be inclined at an angle of 45° to that of reflexion
(that of the paper in the figure), the amplitudes of the
two vibrations will be equal ; and all the conditions
will be fulfilled for the production of circular polarisa-
tion.   Such an instrument was invented by Fresnel,
and is called in consequence Fresnel's rhomb.   On
account of its length and its displacement of the ray,

it is not so convenient as a quarter-undulation plate; but, on the other hand, it affects rays of all wave lengths equally, while the quarter-undulation plate can strictly be adapted to rays of only one wave length.

If either of these instruments be introduced and suitably placed between a selenite plate and the analyser, the chromatic effects will be similar to those due to a plate of quartz cut perpendicularly to the axis.

Another important property of these instruments consists in their effect upon circularly polarised light. Such light may be considered as arising from two plane-polarised rays whose vibrations are perpendicular to one another, and which present a difference of phase equal to a quarter of a wave length. If, therefore, either a quarter-undulation plate or a Fresnel's rhomb be suitably placed, it will either increase or diminish the difference of phase by a quarter of a wave length. In the one case the difference of phase will amount to a half-wave length, in the other it will vanish. And in either case the vibration will be converted into a rectilinear one; but the directions of vibration in the two cases will be perpendicular to one another.

Some years ago Sir Charles Wheatstone devised a very beautiful application of reflexion from a metallic surface for converting plane into circular, or, as he termed it, successive, polarisation. The following is a sketch of the principle upon which it is based. If a ray of plane-polarised light fall upon a metallic reflector, it is divided into two, whose vibrations are respectively parallel and perpendicular to the reflector; and the latter is retarded behind the former by a difference of phase depending upon the angle of
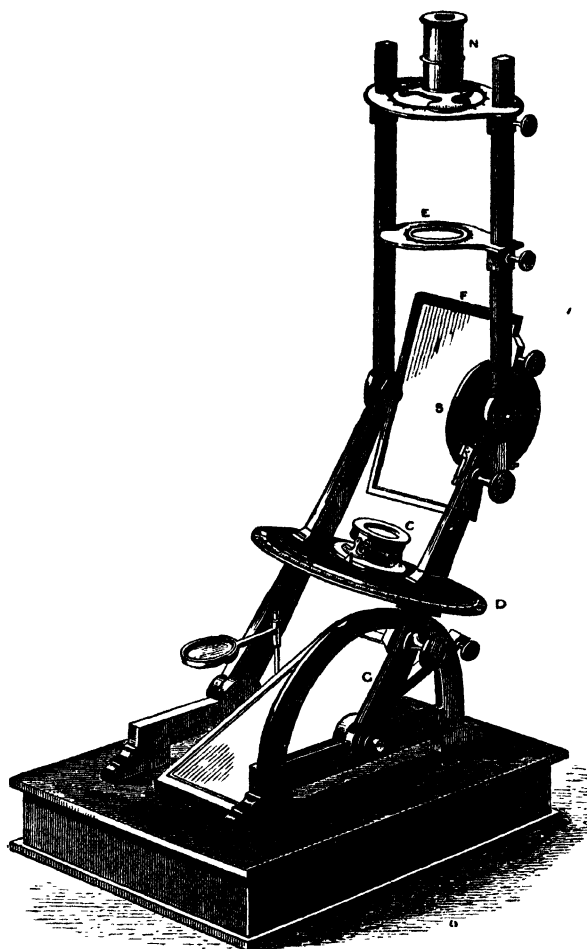
FIG. 15.

incidence. If the plane of vibration of the incident ray be inclined to the plane of incidence at an angle (nearly 45°) which varies with the metal employed, but which is perfectly definite, the intensities become equal. And further, if the angle of incidence have a particular value, dependent upon the nature of the metal (for silver 72°), the retardation will amount to a quarter of a wave length. And the result will be a circularly polarised ray, as in the case of total reflexion.

To give practical effect to the use of this principle, the author of it modified the construction of Norremberg's polariscope so as to admit of the introduction of a silvered plate in a proper position. The following description is quoted from his paper on the subject (*see* Fig. 15).

"A plate of black glass, G, is fixed at an angle of 18° to the horizon. The film to be examined is to be placed on a diaphragm, D, so that the light reflected at the polarising angle from the glass plate shall pass through it at right angles, and, after reflexion at an angle of 18° from the surface of a polished silver plate S, shall proceed vertically upwards. N is a Nicol's prism, or any other analyser, placed in the path of the second reflexion. The diaphragm is furnished with a ring, movable in its own plane, by which the crystallised plate to be examined may be placed in any azimuth. C is a small movable stand by means of which the film to be examined may be placed in any azimuth and at any inclination ; for the usual experiments this is removed.

"If a lamina of quartz cut parallel to the axis, and sufficiently thin to show the colours of polarised light,

be placed upon the diaphragm so that its principal
section (*i.e.* the section containing the axis) shall be
45° to the *left* of the plane of reflexion, on turning
the analyser from left to right, instead of the alterna-
tion of two complementary colours at each quadrant,
which appear in the ordinary polarising apparatus, the
phenomena of successive polarisation, exactly similar
to those exhibited in the ordinary apparatus by a
plate of quartz cut perpendicularly to the axis, will
be exhibited ; the colours follow in the order R, O, Y,
G, B, P, V, or, in other words, ascend as in the case of
a right-handed plate of quartz cut perpendicularly to
the axis.   If the lamina be now either inverted, or
turned in its own plane 90°, so that the principal
section shall be 45° to the right of the plane of re-
flexion, the succession of the colours will be reversed,
while the analyser moves in the same direction as
before, presenting the same phenomena as a left-
handed plate of quartz cut perpendicularly to the axis.
Quartz is a positive doubly refracting crystal ; and in
it consequently the ordinary index of refraction is
smaller than the extraordinary index.   But if we
take lamina of a negative crystal, in which the ex-
traordinary index is the least, as a film of Iceland
spar split parallel to one of its natural cleavages, the
phenomena are the reverse of those exhibited by
quartz : when the principal section is on the *left* of
the plane of reflexion the colours descend, and when
it is on the *right* of the same plane the colours ascend,
the analyser being turned from left to right.

   " It has been determined that the ordinary ray,
both in positive and negative crystals, is polarised in

the principal section, while the extraordinary ray is polarised in the section perpendicular thereto. It is also established that the index of refraction is inversely as the velocity of transmission. It follows from the above experimental results, therefore, that when the resolved ray whose plane of polarisation is to the left of the plane of reflexion is the quickest, the successive polarisation is right-handed, and when it is the slowest, the successive polarisation is left-handed—in the order R, O, Y, G, B, P, V, and in the second case in the reversed order.

"The rule thus determined is equally applicable to laminæ of bi-axal crystals.

"As selenite (sulphate of lime) is an easily procurable crystal, and readily cleavable into thin laminæ capable of showing the colours of polarised light, it is most frequently employed in experiments on chromatic polarisation. The laminæ into which this substance most readily splits contain in their planes the two optic axes ; polarised light transmitted through such laminæ is resolved in two rectangular directions, which respectively bisect the angles formed by the two optic axes : the line which bisects the smallest angle is called the intermediate section ; and the line perpendicular thereto which bisects the supplementary angle is called the supplementary section. These definitions being premised, if a film of selenite is placed on the diaphragm with its intermediate section to the left of the plane of reflexion, the successive polarisation is direct or right-handed ; if, on the contrary, it is placed to the right of that plane, the successive polarisation is left-handed. The ray polarised in the intermediate section is therefore the most

'

retarded ; and as that section is considered to be equi-
valent to a single optic axis, the crystal is positive.

" In one kind of mica the optic axes are in a plane
perpendicular to the laminæ.   They are inclined $22\frac{1}{2}°$
on each side of the perpendicular within the crystal,
but, owing to the refraction, are seen respectively at
an angle of $35°·3$ therefrom.   The principal section is
that which contains the two optic axes.   If the film
is placed on the diaphragm with its principal section
inclined 45° to the left of the plane of reflexion, the
successive polarisation is right-handed.   The ray,
therefore, polarised in the section which contains the
optic axes is the one transmitted with the greatest
velocity.

"Films of uni-axal crystals, whether positive or
negative, and of bi-axal crystals, all agree therefore
in this respect :—that if the plane of polarisation of
the quickest ray is to the left of the plane of reflexion,
the successive polarisation is right-handed when the
analyser moves from left to right ; and if it is to the
right of the plane of reflexion, other circumstances
remaining the same, the successive polarisation is
left-handed.

" It must be taken into consideration that the
principal section of the film is inverted in the reflected
image ; so that if the plane of polarisation of the
quickest ray in the film is to the left of the plane
of reflexion, it is to the right of that plane in the
reflected image.

" It may not be uninteresting to state a few obvious
consequences of this successive polarisation in doubly
refracting laminæ, right-handed and left-handed ac-

cording to the position of the plane of polarisation of
the quickest ray.   They are very striking as experi-
mental results, and will serve to impress the facts
more vividly on the memory.

" 1. A film of uniform thickness being placed on
the diaphragm with its principal section 45° on either
side of the plane of reflexion, when the analyser is at
0° or 90° the colour of the film remains unchanged,
whether the film be turned in its own plane 90°, or be
turned over so that the back shall become the front
surface ; but if the analyser be fixed at 45°, 135°,
225°, or 315°, complementary colours will appear when
the film is inverted from back to front, or rotated in
its own plane either way 90°.

" 2. If a uniform film be cut across and the divided
portions be again placed together, after inverting one
of them, a compound film is formed, which when
placed on the diaphragm, exhibits simultaneously both
right-handed and left-handed successive polarisation.
When the analyser is at 0° or 90° the colour of the
entire film is uniform ; as it is turned round the tints
of one portion ascend, while those of the other
descend ; and when the analyser is at 45° or $n90° +$
45°, they exhibit complementary colours.

" 3. A film increasing in thickness from one edge
to the other is well suited to exhibit at one glance the
phenomena due to films of various thicknesses.   It is
well known that such a film placed between a polariser
and an analyser will show, when the two planes are
parallel or perpendicular to each other and the prin-
cipal section of the film is intermediate to these two
planes, a series of parallel coloured bands, the order

of the colours in each band from the thick towards
the thin edge being that of their refrangibilities, or R,
O, Y, G, B, P, V.   The bands seen when the planes are
perpendicular are intermediate in position to those
seen when the planes are parallel ; on turning round
the analyser these two systems of bands alternately
appear at each quadrant, while in the intermediate
positions they entirely disappear.

  " Now let us attend to the appearances of these
bands when the wedge-form film is placed on the
diaphragm of the instrument, Fig. 15. As the analyser
is moved round, the bands advance toward or recede
from the thin edge of the wedge without any changes
occurring in the colours or intensity of the light, the
same tint occupying the same place at every half-revo-
lution of the analyser.   If the bands advance toward
the thin edge of the wedge, the successive polarisation
of each point is left-handed ; and if they recede from
it, the succession of colours is right-handed ; every cir-
cumstance, therefore, that with respect to a uniform
film changes right-handed into left-handed successive
polarisation, in a wedge of the same substance, trans-
forms receding into advancing bands, and *vice versâ*.
These phenomena are also beautifully shown by con-
cave or convex films of selenite or rock-crystal, which
exhibit concentric rings contracting or expanding in
accordance with the conditions previously explained.

  " 4. Few experiments in physical optics are so
beautiful and striking as the elegant pictures formed
by cementing laminæ of selenite of different thicknesses
(varying form $\frac{1}{20000}$ to $\frac{1}{30}$ of an inch) ·between two
plates of glass. Invisible under ordinary circumstances,

they exhibit, when examined in the usual polarising apparatus, the most brilliant colours, which are complementary to each other in the two rectangular positions of the analyser. Regarded in the instrument, Fig. 13, the appearances are still more beautiful ; for, instead of a single transition, each colour in the picture is successfully replaced by every other colour.   In preparing such pictures it is necessary to pay attention to the direction of the principal section of each lamina when different pieces of the same thickness are to be combined together to form a surface having the same uniform tint ; otherwise in the intermediate transitions the colours will be irregularly disposed.

" 5. A plate of rock-crystal cut perpendicular to the axis loses its successive polarisation, and behaves exactly as an ordinary crystallised film through which rectilinear polarised light is transmitted.

" By means of the phenomena of successive polarisation it is easy to determine which is the thicker of two films of the same crystalline substance.   Place one of the films on the diaphragm E of the instrument (Fig. 15) in the position to show, say, right-handed polarisation'; then cross it with the other film ; if the former be the thicker, the successive polarisation will be still right-handed ; if both be equal, there will be no polarisation ; and if the cross film be the thicker, the successive polarisation will be left-handed.   In this manner a series of films may be readily arranged in their proper order in the scale of tints.

" In the experiments I have previously described the planes of reflexion of the polarising mirror and of the silver plate were coincident ; some of the results

obtained when the azimuth of the plane of reflexion of the silver plate is changed are interesting.

"I will confine my attention here to what takes place when the plane of reflexion of the silver plate is 45° from that of the polarising reflector.

"When the principal sections of the film are parallel and perpendicular to the plane of reflexion of the polarising mirror, as the whole of the polarised light passes through one of the sections, no interference can take place, and no colour will be seen, whatever be the position of the analyser.   •

"When the principal sections of the film are parallel and perpendicular to the plane of reflexion on the silver plate, they are 45° from the plane of reflexion of the polarising mirror.

"The polarised ray is then resolved into two components polarised at right angles to each other ; one component polarised in the plane of reflexion of the silver plate, the other perpendicular thereto ; and one is retarded upon the other by a quarter of an undulation.

"When the analyser is 0° or 90° no colours are seen, because there is no interference ; but when it is placed at 45° or 135°, interference takes place, and the same colour is seen as if light circularly polarised had been passed through the film. The bisected and inverted film shows simultaneously the two complementary colours.

"But when the film is placed with one of its principal sections 22½° from the plane of reflexion of the polarising mirror, on turning round the analyser the appearances of successive polarisation are reproduced, exactly as when the planes of reflexion of the silver

plate and of the polarising mirror coincide. In this case the components of the light oppositely polarised in the two sections are unequal, being as cos $22\frac{1}{2}°$ to sin $22\frac{1}{2}°$; these components respectively fall $22\frac{1}{2}°$ from the plane of reflexion of the silver plate and from the perpendicular plane, and are each resolved in the same proportion in these two planes. The weak component of the first, and the strong component of the second, are resolved into the normal plane, while the strong component of the first and the weak component of the second are resolved into the perpendicular plane.

"The apparatus (Fig. 15) affords also the means of obtaining large surfaces of uncoloured or coloured light in every state of polarisation—rectilinear, elliptical, or circular.

"It is for this purpose much more convenient than a Fresnel's rhomb, with which but a very small field of view can be obtained. It must, however, be borne in mind that the circular and elliptical undulations are inverted in the two methods : in the former case they undergo only a single, in the latter case a double reflexion.

"For the experiments which follow, the crystallised plate must be placed on the diaphragm E, between the silver plate and the analyser, instead of, as in the preceding experiments, between the polariser and the silver plate.

"By means of a moving ring within the graduate circle D the silver plate is caused to turn round the reflected ray, so that, while the plane of polarisation of the ray remains always in the plane of reflexion of the glass plate, it may assume every azimuthal position

with respect to the plane of reflexion of the silver plate. The film to be examined and the analyser move consentaneously with the silver plate, while the polarising mirror remains fixed.

"In the normal position of the instrument the ray polarised by the mirror is reflected unaltered by the silver plate ; but when the ring is turned to 45°, 135°, 225°, or 315°, the plane of polarisation of the ray falls 45° on one side of the plane of reflexion of the silver plate, and the ray is resolved into two others, polarised respectively in the plane of reflexion and the perpendicular plane, one of which is retarded on the other by a quarter of an undulation, and consequently gives rise to a circular ray, which is right-handed or left-handed according to whether the ring is turned 45° and 225°, or 135° and 315°. When the ring is turned so as to place the plane of polarisation in any intermediate position between those producing rectilinear and circular light, elliptical light is obtained, on account of the unequal resolution of the ray into its two rectangular components.

"Turning the ring of the graduated diaphragm from left to right, when the crystallised film is between the silver plate and the analyser, occasions the same succession of colours for the same angular rotation as rotating the analyser from right to left when the instrument is in its normal position and the film is between the polariser and the silver plate."

The same principles apply to the case of bi-axal crystals cut parallel to a plane containing the two optic axes. A ray of plane-polarised light transmitted through such a plate is divided into two, whose

vibrations respectively bisect the angles formed by the two axes. As mentioned above, the line which bisects the smallest angle is called the intermediate section, and the line perpendicular to it the supplementary section ; and the order of the colours depends upon the relative velocity of the two rays. In selenite, the ray whose vibrations lie in the supplementary section is the slowest ; in mica it is the swiftest. Hence these two crystals, all other circumstances being alike, give the colours in opposite orders, and may be regarded as positive and negative, like quartz and Iceland spar. And a test similar to that indicated for uni-axal may be applied to bi-axal crystals.

Some interesting and varied experiments may be made by using two circularly polarising instruments, *e.g.* two quarter-undulation plates (say the plates A and B), one between the polariser proper and the crystal (C) under examination, the other between the crystal and the analyser. The light then undergoes the following processes. If the plate A be placed so that its axis is at 45° on one side or other of the original plane of vibration, and the plate B with its axis perpendicular to that of A, then on turning the analyser we shall have the phenomena of circular polarisation described above. Again, if, the plates A and B retaining the positions before indicated, the crystal C be turned round in its own plane, then, since the light emerging from A and B is circularly polarised, it has lost all trace of direction with reference to the positions of polariser and analyser, and consequently no change will be observed.

Again, if the plates A and B have their axes

directed at 45° on either side of the axis of C, and the three plates be turned round as one piece, the colour will remain unchanged, while if the analyser be turned the colours will follow in the regular order. If the plates A and B have their axes directed at 45° on the same side of the axis of C, and the pieces be turned round bodily as before, the colours change in the same order as above, and go through their cycle once in every right angle of rotation ; and if the analyser be turned in the same direction, the colours change, but in the reverse order. The explanation of this is to be found in the fact that when the plates A and B are crossed, the retardation due to A is compensated by that due to B ; so that the only effective retardation is that due to the crystal C. But upon the latter depends the rotation of the plane of vibration ; if, therefore, the polariser and analyser remain fixed, the colour will remain unaltered. When the plates A and B have their axes parallel, there is no compensation, and the colour will consequently change. It should be added that the rotation of the plane of vibration, and consequently the sequence of colours, does not follow exactly the same law in these cases as in quartz.

Mention was made above of the bands produced in the spectra of polarised light. Beside the fact of the existence of these bands, it has been found upon examination that the state of polarisation at different parts of the interval between two successive bands varies ; and such an examination may be made by means of a quarter-undulation plate or a Fresnel' rhomb.

If we carefully examine the spectrum of light which has passed through a selenite, or other ordinary crystal, placed with its axis at 45° to the plane of vibration, we shall find on turning the analyser that, commencing with two consecutive bands in position, the parts occupied by the bands and those midway between them are plane-polarised, for they become alternately dark and bright; while the intermediate parts, *i.e.* the parts at one-fourth of the distance from one band to the next, remain permanently bright. These are, in fact, circularly polarised. But it would be incorrect to conclude from this experiment alone that such is really the case, because the same appearance would be seen if those parts were unpolarised, *i.e.* in the condition of ordinary light. And on such a supposition we should conclude, with equal justice, that the parts on either side of the parts last mentioned (*i.e.* the parts separated by one-eighth of the interval between two bands) were partially polarised. But if we introduce a quarter-undulation plate between the selenite and analyser, with its axis inclined at 45° to that of the selenite, circular polarisation will be converted into plane and plane into circular. This being so, the parts which were originally banded ought to become bright and to remain bright, while those that were originally bright ought to become banded during the rotation of the analyser. The effect to the eye will consequently be a general shifting of the bands through one-fourth of the space which separates each pair. Further, as on the one hand plane polarisation is converted into circular right-handed or left-handed by two positions of the

plate at right angles to one another, so on the other
right-handed circular polarisation will be converted
by the plate in a given position into plane polarisa-
tion having the vibrations in one direction, and left-
handed into plane polarisation having the vibrations
in a direction at right angles to the former. Hence,
if the plate be turned through a right angle from the
position first described, the bands will be shifted in a
direction opposite to that in which they were moved
at first. In this we have evidence not only that the
polarisation on either hand is circular, but also that
on the one side it is right-handed, while on the other
it is left-handed. A similar train of reasoning will
show that the parts intermediate between the plane
and the circularly polarised portions are elliptically
polarised.

CHAPTER VI.

PHENOMENA PRODUCED BY MECHANICAL MEANS—UNANNEALED GLASS.

ALL the phenomena hitherto described manifestly depend upon the internal structure of the crystal plate, in virtue of which it affects the vibratory movement of the ether within it differently in different directions.   And seeing that most crystals, when broken, divide themselves naturally into smaller crystals having the same form, *i.e.* having their planes and edges similarly inclined, we are naturally led to conclude that the structure of these bodies may differ not so much in different parts, as along different lines or planes connected with the forms into which they break, or (as it is also described) with their planes of natural cleavage.   And this suggests the question whether an uncrystalline body might not, by pressure, or strain, or other mechanical distortion, be caused to affect the motions of the ether within it in a manner dependent upon their direction, and in that way to exhibit chromatic effects with polarised light analogous to those described above.   Experiment answers this question in the affirmative.

The simplest experiment in this branch of inquiry consists in taking a rectangular bar of ordinary glass ; and having crossed the polariser and analyser so as to give a dark field, to strain the bar with both hands as if we were trying to bend it or to break it across. The side towards which it may be supposed to be bent is of course compressed, while the opposite is stretched out.   Between these two there must be an intermediate band, more or less midway between the two, which is neither compressed nor stretched.   The moment the strain is put upon the bar, light will be seen to pass through the part of the bar nearest to both sides, while a band remains dark midway between the two.   This shows that the mechanical strain has
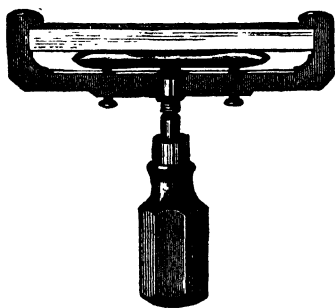
FIG. 16.

imparted to portions of the glass a structural cha- racter analogous, at all events optically, to that of a crystal.

The effects may be increased and rendered more striking by placing the glass in a frame furnished with a screw, by which the rod may be firmly held

and considerable pressure applied at particular points
(Fig. 16). When this is done the structural character
becomes more completely developed, and the dark
band is fringed with colours which appear to flow
inwards or outwards according as the pressure is
increased or diminished. A slightly different, but
more effective, exhibition of chromatic polarisation is
produced by squeezing a thick square plate of glass in
a vice. In this case the pressure may be carried fur-
ther without fear of fracture, and the chromatic effects
heightened (Fig. 17).

FIG. 17.

It is, however, well known that molecular forces,
such as those due to heat and cooling, in many cases
far transcend in intensity those which we can exert

by mechanical arrangements. And, in fact, if a block of glass be unequally heated to a very moderate degree, the internal structural effects immediately reveal themselves by dark bands, which serve as indications of strain and pressure. As the block cools, these landmarks gradually disappear, and the field becomes again uniformly dark. But by far the most splendid effects (and these are permanent) are produced by unannealed glass ; that is, by glass which has been rapidly and therefore unequally cooled. When a mass of glass has been cast in a mould in the form of a thick plate, then, whatever be the contour line, the outside will cool first and become a rigid framework, to which the interior of the mass must accommodate itself. The nature and direction of the pressure at each point of the interior will be primarily dependent upon the form of the contour ; and by adopting various forms of contour the most beautiful and varied figures with coloured compartments may be produced. Among the dark bands some will be found to retain their position while the glass is turned round ; these answer to the systems of isochromatic curves in uni-axal and bi-axal crystals (explained below, Chapter VIII.), and mark the borders between regions of strain and of pressure. Beside these there are also variable bands, which correspond to the brushes which cross the isochromatic curves. The forms and colours of the figures produced by transparent bodies when submitted to polarised light have been conversely used as a means of measuring, with almost unparalleled accuracy, the mechanical pressures which such a body is undergoing.

CHAPTER VII.

*ATMOSPHERIC AND OTHER POLARISATION—
THE POLAR CLOCK.*

WE may here digress with advantage for a few
moments from the direct course of our subject, in
order to notice some instances of the phenomena of
polarisation which occur in everyday experience, as
distinguished from laboratory experiments and in-
strumental contrivances. But the reader will doubt-
less remember that to these, as well as to all other
cases, the remark made at the outset applies, viz.
that instrumental aid must be invoked in order to
render them perceptible by our eyes. *

Besides glass, many other substances may be used
as reflectors so as to produce polarisation ; for example,
leaves of trees, particularly ivy, mahogany furniture,
windows, shutters, and often roofs of houses, oil paint-
ings, &c., and last, but not least, the surface of water.
In each of these cases, when the reflected beam is ex-
amined with a Nicol, the alternations of light and
darkness are most strongly marked, and the colours
(if a crystal plate be used) are most vivid, or in tech-
nical language the polarisation is most complete, when

the light is reflected at a particular angle. In proportion as the inclination of the incident light deviates from this angle, the colours become fainter, until, when it deviates very greatly, all trace of polarisation disappears.

The minor practical applications of these facts are numerous, and several may suggest themselves to the reader, especially if he carries a Nicol and a quartz plate in his pocket. Among them the following may be mentioned :—A Nicol so placed as to extinguish superficial reflexion or glare will often be found of considerable use in examining oil pictures placed inconveniently in respect of light. Again, it has been suggested that a similar contrivance might be used by persons engaged in the sport of fishing, in order the better to see the fish or other objects at a moderate depth below the surface of the water. A Nicol's prism has also been placed with very good effect in the eyepiece of a telescope (the precise disposition depending upon the internal construction of the instrument), in order to diminish the intensity of sunlight. This arrangement has the additional advantage of reducing the heat also, because the rays of heat are affected in respect of polarisation very nearly in the same way as those of light.

It will be found very interesting to examine the polarisation of sunshine reflected from ripples on the surface of a lake, or better still from the waves of the sea, and its different degrees of completeness produced at the variously inclined portions of the waves. But, without having recourse to Nature on so large a scale, an artificial piece of water may be placed in our room.

A tea-tray will serve as well as anything else to form our little sea; and a periodic tap at one corner will cause ripple enough for the present purpose. The waves appear bright, and although brighter in some parts than in others, they are nowhere entirely dark. But on turning the Nicol round, the contrast of light and darkness becomes much stronger than before. In parts the light is absolutely extinguished, or the polarisation is complete; in others it is incomplete in various degrees. And if a selenite or other crystal plate be introduced, we have the beautiful phenomena of iris-coloured rings playing over the surface of our miniature sea.

Suppose that we now turn our attention to the sky, and on a clear bright day we sweep the heavens with a polariscope, or even with a mere Nicol's prism; we shall find traces of polarisation in many directions. But if we observe more closely we shall find that the most marked effects are produced in directions at right angles to a line drawn from our eye to the sun—when, in fact, we are looking across the direction of the solar beams. Thus, if the sun were just rising in the east or setting in the west, the line of most vivid effect would lie on a circle traced over the heavens from north to south. If the sun were in the zenith, or immediately overhead, the most vivid effects would be found on the horizon; while at intermediate hours the circle of strongest polarisation would shift round at the same rate as the shadow on a sun dial, so as always to retain its direction at right angles to that of a line joining ourselves and the sun.

Now, what is it that can produce this effect, or,

indeed, what produces the effect of light from all parts of a clear sky ?   The sky is pure space, with no contents, save a few miles of atmosphere of the earth, and beyond that the impalpable ether, supposed to pervade all space, and to transmit light from the furthest limits of the stellar universe. The ether is, however, certainly inoperative in the diffusion of light now under consideration.   But a very simple experiment will suffice to show that such a diffusion, or, as it has been better called, a scattering of light, is due to the presence of small particles in the air.   If a beam from an electric lamp or from the sun be allowed to pass through a room, its track becomes visible by its reflexion from the motes of floating bodies—in fact, by the dust in the air.   But if the air be cleared of dust by burning it with a spirit lamp placed underneath, the beam disappears from the parts so cleared, and the space becomes dark.   If, therefore, the air were absolutely pure and devoid of matter foreign to it, the azure of the sky would no longer be seen and the heavens would appear black; the illumination of objects would be strong and glaring on one side, and on the other their shadows would be deep and unrelieved by the diffused light to which we are accustomed.   Now, beside the dust, there are always minute particles of water floating in the atmosphere.   These vary in size from the great. rain drops which fall to earth on a sultry day, through intermediate forms of mist and of fine fleecy cloud, to almost invisible minuteness. And it is some of these floating particles—whether of water or other matter is perhaps doubtful—which scatter the solar rays and suffuse the heavens with

light. Now it is a remarkable fact, established by Professor Tyndall, while operating with minute traces of gaseous vapours, that while coarser particles scatter rays of every colour—in other words, scatter white light—finer particles scatter fewer rays from the red end of the spectrum, while the finest scatter only those from the blue end. And in accordance with this law clouds are white, clear sky is blue.

But the point which most concerns us here is the fact, also discovered by Professor Tyndall, that light scattered laterally from fine particles is polarised. The experiment by which this is most readily shown is as follows :—Allow a beam of solar or other strong light to pass through a tube about thirty inches long, filled with water with which a few drops of mastic dissolved in alcohol have been mixed. The fluid so formed holds fine particles of mastic in a state of suspension, which scatter the light laterally ; and if the scattered light be examined with a Nicol, traces of polarisation will be detected. But, better still, as suggested by Professor G. G. Stokes, instead of using the scattering particles as a polariser, and the Nicol as an analyser, we may polarise the light before it enters the tube, and use the particles as an analyser, and thus produce the same effect as before, not only upon the particular point of the beam to which the eye is directed, but upon the whole body of scattered light. As the Nicol is turned, the light seen laterally begins to fade ; and when the instrument has been turned so as to cut off all vertical vibrations, the only parts remaining visible in a horizontal direction will be those reflected from the larger impurities floating in the water indepen-

dently of the mastic. The direction of vibration of
the light polarised by lateral scattering is easily re-
membered by the fact that the vibrations must be
perpendicular both to the original and to the scat-
tered beam ; if, therefore, the latter be viewed hori-
zontally, they must be perpendicular to two horizontal
straight lines at right angles to one another, *i.e.* they
must be vertical.

An effect still more beautiful, and at the same
time, perhaps, more instructive, may be produced by
interposing a plate of quartz between the Nicol and
the tube. The whole beam then becomes suffused
with colour, the tint of which changes for a given
position of the spectator with the angle through which
the Nicol is turned.

And not only so, but while the Nicol remains at
rest the tints are to be seen scattered in a regular and
definite order in different directions about the sides
of the beam.   But this radial distribution of colours
may also be shown in a more striking manner by
using a biquartz, which, as explained before, distri-
butes the colours in opposite directions. The beam
should in every case be viewed at right angles ; the
more obliquely it is viewed the less decided is the
polarisation.

The colours here seen are those which would be
observed upon examining a clear sky in a position
90° from that of the sun ; and the exact tint visible
will depend upon the position in which the Nicol is
held, as well as upon that of the sun.   Suppose, there-
fore, that a Nicol and quartz plate be directed to
that part of the sky which is all day long at right

angles to the sun—that is, to the region about the
north pole of the heavens (accurately to the north
pole at the vernal and autumnal equinox)—then, if
on the other hand the Nicol be turned round, say in
a direction opposite to that of the sun's motion, the
colours will change in a definite order; if, on the.
other, the Nicol remain stationary while the sun moves
round, the colours will change in a similar manner.
And thus in the latter case we might conclude the
position of the sun, or, in other words, the time of the
day, by the colours so shown.   This is the principle
of Sir Charles Wheatstone's polar clock, which is one
of the few practical applications which this branch of
polarisation has yet found.

Figs. 18 and 19 represent general forms of this
instrument described in the following passage by the
. inventor :—

"At the extremity of a vertical pillar is fixed,
within a brass ring, a glass disc, so inclined that its
plane is perpendicular to the polar axis of the earth.
On the lower half of this disc is a graduated semicircle,
divided into twelve parts (each of which is again sub-
divided into five or ten parts), and against the divi-
sions the hours of the day are marked, commencing
and terminating with VI.   Within the fixed brass
ring, containing the glass dial-plate, the broad end of
a conical tube is so fitted that it freely moves round
its own axis; this broad end is closed by another
glass disc, in the centre of which is a small star or
other figure, formed of thin films of selenite, exhibit-
ing, when examined with polarised light, strongly
contrasted colours; and a hand is painted in such a

position as to be a prolongation of one of the principa
sections of the crystalline films.   At the smaller en
of the conical tube a Nicol's prism is fixed so tha
either of its diagonals shall be 45° from the principa
section of the selenite films.   The instrument bein
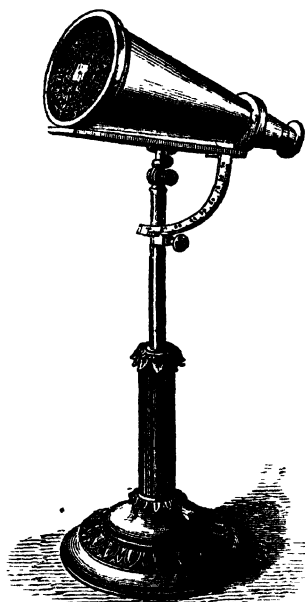
FIG. 18.—Wheatstone's Polar
Clock.
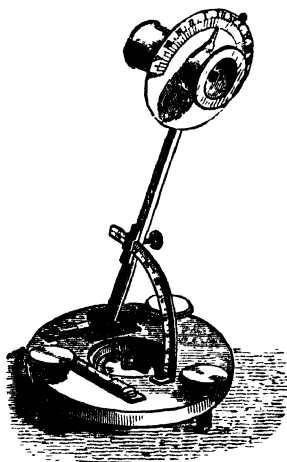
FIG. 19.—Wheatstone's Pola
Clock.

so fixed that the axis of the conical tube shall coi
cide with the polar axis of the earth, and the eye
the observer being placed to the Nicol's prism, it w
be remarked that the selenite star will in general
richly coloured ; but as the tube is turned on its ax

the colours will vary in intensity, and in two positions will entirely disappear. In one of these positions a smaller circular disc in the centre of the star will be a certain colour (red, for instance), while in the other position it will exhibit the complementary colour. This effect. is obtained by placing the principal section of the small central disc 22½° from that of the other films of selenite which form the star. The rule to ascertain the time by this instrument is as follows :— The tube must be turned round by the hand of the observer until the coloured star entirely disappears, while the disc in the centre remains red ; the hand will then point accurately to the hour. The accuracy with which the solar time may be indicated by this means will depend on the exactness with which the plane of polarisation can be determined ; one degree of change in the plane corresponds with four minutes of solar time.

"The instrument may be furnished with a graduated quadrant for the purpose of adapting it to any latitude ; but if it be intended to be fixed in any locality, it may be permanently adjusted to the proper polar elevation, and the expense of the graduated quadrant be saved : a spirit-level will be useful to adjust it accurately. The instrument might be set to its proper azimuth by the sun's shadow at noon, or by means of a declination needle ; but an observation with the instrument itself may be more readily employed for this purpose. Ascertain the true solar time by means of a good watch and a time equation table, set the hand of the polar clock to correspond thereto, and turn the vertical pillar on its axis until the colours of

the selenite star entirely disappear.   The instrumen·
then will be properly adjusted.

"The advantages a polar clock possesses over ?
sun-dial are :—1st. The polar clock being constantl)
directed to the same point of the sky, there is no localit)
in which it cannot be employed, whereas, in order tha·
the indications of a sun-dial should be observed durin₤
the whole day, no obstacle must exist at any tirṁ
between the dial and the places of the sun, and i
therefore cannot be applied in any confined situation
The polar clock is consequently applicable in place·
where a sun-dial would be of no avail—on the nortҺ
side of a mountain or of a lofty building, for instance
2ndly. It will continue to indicate the time after sunseٰ
and before sunrise—in fact, so long as any portion o
the rays of the sun are reflected from the atmosphere
3rdly. It will also indicate the time, but with les·
accuracy, when the sky is overcast, if the clouds do noٰ
exceed a certain density.

"The plane of polarisation of the north pole of thٰ
sky moves in the opposite direction to that of the hanٰ
of a wåtch ; it is more convenient therefore to have thٰ
hours graduated on the lower semicircle, for the figure·
will then be read in their direct order, whereas the)
would be read backwards on an upper semicircle.   Iٰ
the southern hemisphere the upper semicircle shoulٰ
be employed, for the plane of polarisation of the soutҺ
pole of the sky changes in the *same* direction as th
hand of a watch.   If both the upper and lower semi
circles be graduated, the same instrument will serv
equally for both hemispheres."

The following is a description of one among severа

other forms of the polar clock which have been devised. This (Fig. 20), though much less accurate in its indications than the preceding, beautifully illustrates the principle.

"On a plate of glass twenty-five films of selenite of equal thickness are arranged at equal distances radially in a semicircle ; they are so placed that the line bisecting the principal sections of the films shall correspond with the radii respectively, and figures corresponding
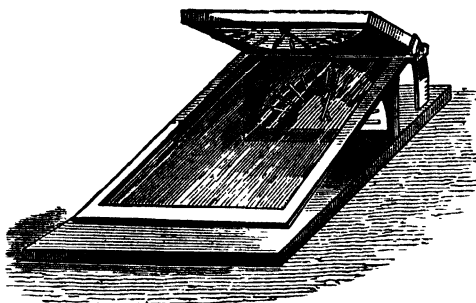
FIG. 20.—Polar Clock.

to the hours are painted above each film in regular order.   This plate of glass is fixed in a frame so that its plane is inclined to the horizon 38° 32', the complement of the polar elevation ; the light, passing perpendicularly through this plate, falls at the polarising angle 56° 45' on a reflector of black glass, which is inclined 18° 13' to the horizon.   This apparatus being properly adjusted—that is, so that the glass dial-plate shall be perpendicular to the polar axis of the earth— the following will be the effects when presented towards an unclouded sky.   At all times of the day

the radii will appear of various shades of two com-
plementary colours, which we will assume to be red
and green, and the hour is indicated by the figure
placed opposite the radius which contains the most
red ; the half-hour is indicated by the equality of two
adjacent tints."

The above remarks have reference to the polarisa-
tion of the sky at a given point, viz. 90° from the
sun, at different times of the day. But the distribu-
tion of the polarisation at different points of the sky
at the same time, a subject of more difficulty, has
been studied by Brewster, and others, and latterly by
Mr. Bosanquet. It appears to have been in the first
place established that, in a plane containing the sun,
the zenith, and the observer (say, the zenith plane),
the direction of polarisation undergoes more than one
change. Suppose, for simplicity, the sun to be in the
horizon ; then, between the sun and a point (called a
neutral point) about 18° 30′ above the horizon, the
plane of polarisation is at right angles to the zenith
plane. Between the neutral point above mentioned
(observed by Babinet) and another neutral point
18° 30′ above the anti-solar point (observed by
Arago) the plane of polarisation coincides with the
zenith plane ; while between Arago's point and the
horizon it is perpendicular to the zenith plane. Out-
side the zenith plane it appears (as established by
Mr. Bosanquet, in correction of Brewster's conclu-
sions, and of maps constructed upon them) that the
tract of the sky, separating regions whose polarisation
is respectively perpendicular and parallel to the
zenith plane, presents traces of polarisation in planes

inclined at various angles to the zenith plane—in other words, the direction of the polarisation changes by a gradual rotation—and that its amount does not pass through zero except at the neutral points.

Among the phenomena of polarised light which may be observed either with a Nicol's prism or even with the naked eye, one of the most curious, and perhaps not yet fully explained, is that of Haidinger's brushes. If the eye receives a beam of polarised light, a pale yellow patch in the form of an hour-glass, the axis of which is perpendicular to the plane of vibration, is perceived. On either side of the neck of the figure two protuberances of a violet tint are also seen to extend. After a little practice these figures or "brushes" may readily be observed. If the day be cloudy, a Nicol must be used and directed to a tolerably bright cloud. The brushes are better defined in one position than in others; but if the Nicol be turned round, the brushes will be seen to revolve with it. If on a clear day we look in a direction at 90° from that of the sun, where the skylight is most completely polarised, the brushes may be seen with a naked eye. The reader should, however, be warned that the appearance of these brushes is not a permanent phenomenon. They are, in fact, visible for only a couple of seconds or so after the polarised light is first received on the eye. But the effect may be revived by changing the plane of polarisation. Helmholtz connects the phenomenon with some double refraction due to the yellow spots in the eye, with the area of which that of the brushes is coincident.

CHAPTER VIII.

*RINGS AND BRUSHES PRODUCED BY CRYSTAL.
PLATES.*

THE phenomena hitherto discussed are those pro-
duced by a beam of parallel rays ; it remains to consider
those due to systems of convergent or divergent rays.
It was seen in the former case that the effect produced
by a plate of crystal upon polarised light depended
partly upon the direction of the ray with reference to
the natural structure (or more particularly to the optic
axis or axes) of the crystal, and partly to the thickness
of crystal traversed by the ray.   And it is clear that if
a system of convergent or divergent rays fall upon a
plate of crystal of uniform thickness, the rays will
strike the plate at various angles, and traverse various
thicknesses of the crystal.   Hence it follows that the
effects produced will not be, as in the case of parallel
rays, fields of uniform tint, but figures of various form
and tint.

It was explained above that in Iceland spar there
is a particular direction, viz. that of the line joining the
two opposite obtuse angles of the natural crystal, in
which there is no double refraction, and in which all
rays travel with the same velocity.   This direction

(that is to say, this line and all lines parallel to it) bears the name of the optic axis.   There are many other crystals having the same property in one and only one direction—in other words, having a single optic axis.   There is, moreover, another class of crystals having two such axes.   Crystals of the first class, or uni-axal crystals, are again divided into two groups, viz. positive, in which the extraordinary ray is more refracted than the ordinary, and negative, in which the ordinary ray is the more refracted.   It will be remembered that the ray which travels slowest is the most refracted. Among the former may be mentioned—

UNI-AXAL CRYSTALS.

Positive.

Cassiterite   Phenakite.
Dioptase.   Quartz.
Ice.   The " Red Silver " ores.
Lead Hyposulphate.   Zircon.
Magnesium Hydrate.

Positive or Negative.

Apophyllite.   Pennine.

Negative.

Anatase.   Idocrase.
Apatite.   Mellite.
Beryl   Mimetesite.
Calcite.   Nepheline.
Cinnabar.   Pyromorphite.
Corundum (ruby and   Tourmaline.
    sapphire).   Wultenite.

Every crystal belongs to one or other of six sys-
tems or types of symmetry. These are conveniently
distinguished by the various kinds of axes to which
the faces are referred for the purpose of geometrical
comparison.

(1.) The regular system, which is based upon a
system of three equal rectangular axes. Any form
derived from this will be perfectly symmetrical with
reference to the three axes, and will present no dis-
tinguishing feature in relation to any of them. Crystals
belonging to this system have no optic axis nor any
doubly refracting property. The cube and regular
octahedron are examples of forms in this system.

(2.) The tetragonal system, based upon a system
of three rectangular axes, whereof two are equal, but
the third greater or less than the other two. Crystals
belonging to this system have one optic axis coin-
ciding with the last-mentioned crystallographic axis.
The square-based prism is a form of this system.

(3.) The hexagonal system, referred by some crys-
tallographers to three equal axes lying in one plane,
inclined·at 60° to one another, and a fourth axis at
right angles to the other three. Crystals of this
system have one optic axis coinciding with the fourth
axis above mentioned. The rhombohedron (the six
faces of which are equal rhombs) is a form of this
system. Iceland spar, or calcite, has cleavages parallel
to the faces of a rhombohedron, the obtuse plane
angles of which are 101° 5′. Some crystallographers
refer the crystals of this system to axes parallel to the
edges of a rhombohedron ; these axes being three in
number obliquely but equally inclined to each other,
and of equal lengths.

(4.) The orthorhombic system, having three rectangular but unequal axes.

(5.) The clinorhombic system, which differs from the rhombic in this, that while one of the three axes is perpendicular to the other two, these two are oblique to one another.

(6.) The anorthic system, in which all the axes are óblique.

All crystals belonging to the last three systems have two optic axes. In the rhombic system they lie in a plane containing two of the three crystallographic axes; in the monoclinic they lie either in the plane containing the oblique axes or in a plane, or for different colours in planes, at right angles thereto. In the triclinic no assignable relation between the optic and the crystallographic axes has been determined.

It was shown above that the retardation due to any doubly refracting crystal, and consequently the colour produced by it, is dependent on the thickness; and that with a crystal of constantly increasing thickness the colours go through a complete cycle, and then begin again. Suppose, then, a divergent beam to fall perpendicularly upon a uni-axal crystal plate cut at right angles to the optic axis; the central rays will fall perpendicularly to the surface; but the rays which form conical shells about that central ray will fall obliquely. The rays forming each shell will fall with the same degree of obliquity on different sides of the central ray, those forming the outer shells having greater obliquity than the inner. Now, the more obliquely any ray falls upon the surface the greater will be the thickness of the crystal which it traverses;

and this will still be the case even though it suffers
refraction, or bending towards the perpendicular, on
entering the crystal.   Each incident cone of rays will
consequently still form a cone when refracted within
the crystal, although less divergent than at incidence,
in its passage through the plate ; and the successive
refracted cones will be more and more oblique, as were
the incident cones, but in a less degree, as we pass
from the more central to the more external members
of the assemblage forming the beam of light.

Now, not only will the actual thickness of crystal
traversed by the ray, but so also will the double
refraction, increase with the obliquity or divergence
of direction from that of the axis ; and the effect of
the former is in fact small compared with that of the
latter in producing retardation.

Let A B C D (Fig. 21) represent the crystal plate,
O P the direction of the optic axis and of the central
ray, O n, O n' those of any two other rays.   The ray O P
will not be divided ; but O n will be separated by the
double refraction of the plate into two, $n s$, $n r$, the one
ordinary, the other extraordinary; and these will emerge
parallel to one another, and may be represented by
the lines $s t$, $r v$.   Similarly, the effect of double re-
fraction on O n' may be represented by $n' s'$, $n' r'$,
$s' t'$, $r' v'$.   Suppose now that the process were re-
versed, and that two monochromatic rays, one ordinary,
the other extraordinary, reach the plate at s and r in
the directions $t s$, $v r$ respectively ; these would meet
at n and travel together to O.   Suppose, further, that
the difference in length of $s n$ and $r n$ is equal to one
wave length ; then, since one of them is an ordinary

and the other an extraordinary ray, their vibrations will be perpendicular to one another ; and if the polariser and analyser be crossed, the point *n* viewed
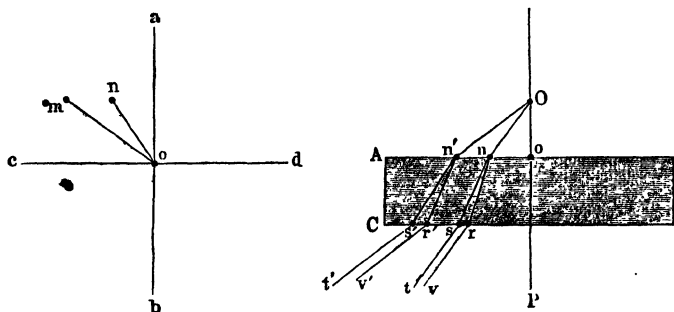
FIG. 21.

from O will appear dark.   Similarly, if two rays arrive in the directions *t′ s′, v′ r′* at the points *s′ r′* respectively, they will meet at *n′* and proceed together to O ; and if the difference of the paths *s′ n′, r′ n′* be two wave lengths, the point *n′* will also appear dark.   A pair of rays reaching the crystal at points •between the pairs before mentioned will emerge at a point *n″* between *n* and *n′*, and will present a difference of phase equal to a half-wave length.   On principles explained in an earlier part of these lectures, such a point *n″* will appear bright.   On either side of *n″*—that is, towards *n* and *n′*—the light will gradually fade.   The same alternations of light and darkness will recur at intervals as we proceed along any straight line drawn outwards from the central ray.   And inasmuch as the obliquity of the ray is the same for every point equi-

distant from the centre O, it follows that the phenomena of light and darkness will be the same throughout each circle drawn about the centre O. In other words, the centre will be surrounded by rings alternately bright and dark. The diameters of the rings depend, as was seen above, on the wave length of the particular light used, and will consequently be different for different coloured rays. If, therefore, white light be used, the different coloured rings would not coincide, but would be disposed in recurring series as we proceed outwards from the centre.

Another effect would, however, also be produced. Suppose the polariser and analyser to be so placed that, the field being regarded as a map, the vibrations in the one being E. and W., those in the other N. and S. ; then of the two rays emerging at the most northern or the most southern point of any ring the vibrations of one would be towards the axis, or N. and S. ; those of the other would be across it, or E. and W. And of these one would be extinguished by the polariser, the other by the analyser ; and the same will be the case for every ring. Hence, throughout a N. and S. line crossing the entire field the light will be extinguished ; and a similar effect will obviously occur along an E. and a W. line. Hence, when the polariser and analyser are crossed, the entire system of rings will be intersected by a black cross, two of whose arms are parallel to the plane of vibration of the polariser, and two to that of the analyser, and the rings in the quadrants on each side of an arm are of complementary tints. When the analyser is turned round through a right angle from its former position,

only one set of vibrations (say those executed in a direction E. and W.) will be extinguished, and consequently along one pair of arms of the cross the ordinary rays will pass undisturbed, along the other the extraordinary; that is to say, the cross will be white. When the polariser and analyser occupy any other position than those noticed above, there are two crosses inclined at an angle equal to that between the planes of vibration, each arm of which separates complementary rings. (*See* Plate, Figs. 1 and 2).

Various forms of polariscopes have been devised for showing the crystal rings. The simplest of these is the tourmalin forceps, which consists of two plates of tourmalin fixed in cork discs; the latter are encircled in wire in such a way that they may be turned round in their own planes. The wire after encircling one disc is bent round so as to form a handle; it then encircles the other; and the elasticity of the wire allows the pair of discs to be opened and shut like a pair of pincers. If a crystal plate be inserted between the two, and the whole held close to the eye, the rays from parts of the field at different distances from the centre will reach the eye, having traversed the crystal with different degrees of obliquity; and a system of rings and brushes will be formed.

Another method consists in applying to Norremberg's polariscope a pair of lenses, one below the crystal with the crystal in the focus, the other above it. The first ensures that the rays shall traverse the crystal with different degrees of obliquity; the second brings within the range of vision rays which would otherwise fail to reach the eye, and at the same time

converging them into a cone with a smaller vertical angle, renders the ring smaller than when seen with the simple tourmalins. An additional lens of greater focal length, *i.e.* of less power, is often added in order to adjust the whole to individual eyesight.
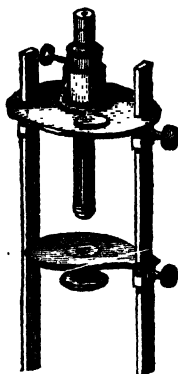
FIG. 22.

Fig. 22 gives the general appearance of the addition to the apparatus of Norremberg described above, and Fig. 23 the course of a system of rays brought to a focus on the crystal by the lens *a b*, and again converged by a second lens *c d.*

But by far the most successful arrangement for enlarging the field of view so as to comprise the complete system of rings even with bi-axal crystals having widely inclined optic axes, is the system of lenses due in the first instance to Norremberg. The disposition of the parts, and the general appearance of the instrument as constructed by Hofmann of Paris, and called by him the "Polarimicroscope," is

FIG. 23.

I

FIG. 24.

$\frac{1}{2}$
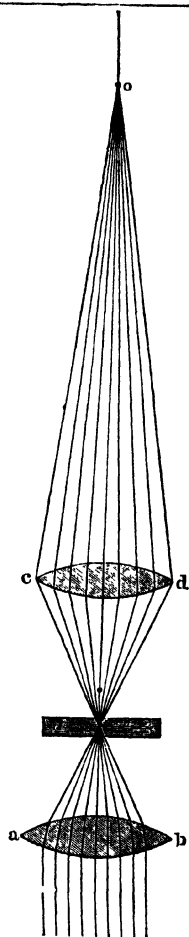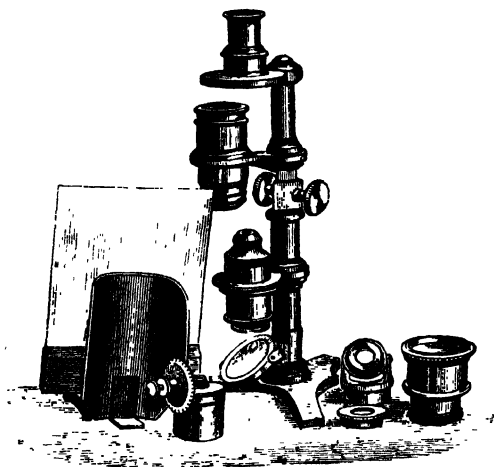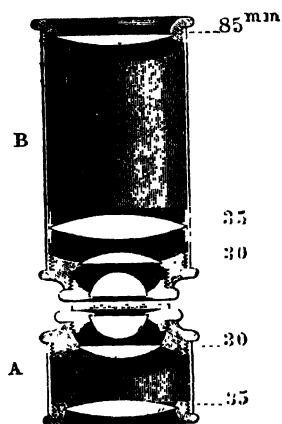
FIG. 25.

given, Fig. 24. The details of the lenses, together
with their focal lengths in millimetres, are represented
half size in Fig. 25. In this instrument the lenses
which converge the rays upon the crystal plate can
be taken out, and replaced by others giving parallel
light ; it can then be used as an ordinary polariscope.

An important modification of this instrument has
been made by Prof. W. G. Adams, in which the prin-
cipal objects in view have been :—

•(1.) To obtain an extensive field of view.

(2.) To afford a means of measuring the rings
and the angles between the optic axes of biaxal
crystals.

(3) To have a means of immersing the crystal in
a liquid in those cases in which the optic axes are
too far apart to be seen in air.

These advantages have been obtained by modi-
fying the positions and focal lengths of the lenses
usually employed in table polariscopes, so that the
rings of a crystal are best seen when there is a space
of 1¼ inch between the two lenses, one on either side
of the crystal. Into this space is introduced a central
piece, consisting of a circular box with deep plano-
convex lenses fixed, one in the bottom and the other
in the top of the box, in such a position that their
curved surfaces have a common centre of curvature,
with their flat faces turned towards one another and
enclosing the crystal between them. The box can
turn about an axis passing through the common
centre of curvature.

The instrument is fully described in the *Philo-
sophical Magazine* for July 1875 ; but the following

diagram (Fig. 25A) extracted, by permission, from the paper will explain the general disposition of the parts.

The advantages to be obtained by the use of the central piece are :—

(1.) The extension of the field of view. If the angle in air corresponding to the field of view is 74° without the central piece, then the angle will be increased to about 128° when the central piece is introduced, the central piece giving the same angle in glass that is given without it in air. The field of view may be made to include both optic axes of topaz of Brazil.

(2.) When the plane containing the optic axes is at right angles to the axis P Q, either of the optic axes of a biaxal crystal or any ring may be brought into the centre of the field of view where spider-lines cross one another, and the angles between them accurately measured.

Mention has been made above of the effect of the circular polarisation of quartz in the colours produced by a beam of parallel rays of polarised light. It will be worth while to examine the modification which the rings and brushes undergo from the same cause. It has been explained that a ray of plane-polarised light in traversing a crystal of quartz in the direction of its axis is divided into two, the vibrations of which are circular, one right-handed, the other left. If the ray traverses the crystal in a direction perpendicular to the axis, and if the original vibrations are neither parallel nor perpendicular to the axis, it is also divided into two, whereof the vibrations are not circular, but rectilinear. It was suggested

FIG. 25A.
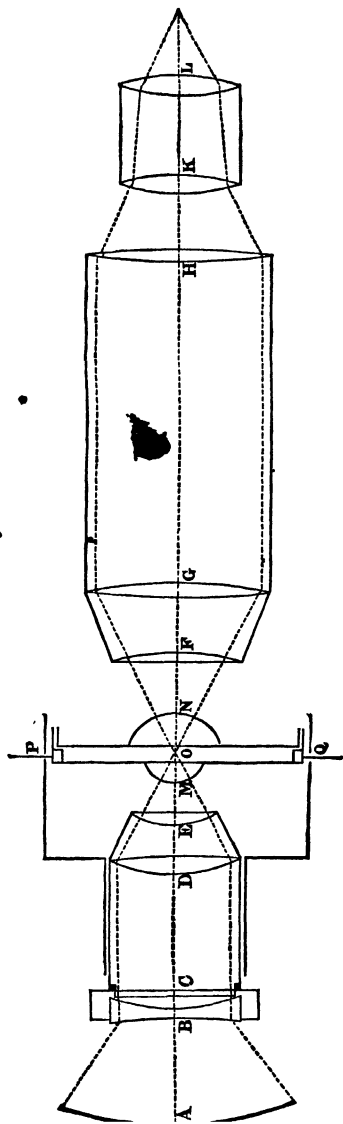
A. Concave mirror.
B. Double-concave lens I inch ... diameter.
C. Tourmaline or other polari... r.
D. Double-convex crossed len I inch in diameter, $1\frac{1}{4}$ inch focal length.
E. Convexo-plane lens $\frac{4}{5}$ inch diameter, I inch focal length.
F. Plano-convex lens I inch ii diameter, $1\frac{1}{4}$ inch focal length.
G. Double-convex crossed len $1\frac{1}{2}$ inch in di...neter, $1\frac{3}{4}$ inch focal length.
H. Double-convex lens $1\frac{3}{4}$ inc in diameter, 3 inches focal length.
K L. Nicol's prism and eyepi... e.
M and N. Portions of hemis... herical lenses with O as the common centre of curvature of their spherical surfaces. Radius of curvature of M ; $\frac{1}{4}$ inch, and of N is $\frac{3}{8}$ of an inch, and the distance between the lenses M and N about $\frac{1}{8}$ of an inch.

P and Q. The ends of the axi... supporting the box fixed so that the axis passes through the point O. The dotted lines show the pat... of the light through the instrument.

first by Sir G. Airy, that these circular and rectilinear
vibrations are limiting cases of elliptical; and both
theory and experiment tend to confirm the suggestion,
by showing that if the ray be incident on the crystal
in any direction oblique to the axis, it is divided into
two, the vibrations of which are similar ellipses
having the longer diameter of the one coincident
with the shorter of the other, and the motion in
the two oppositely directed.   The longer diameters
of the ellipses coincide with the directions of vibra-
tion of the ordinary and extraordinary rays in the
case of an ordinary positive crystal; and are con-
sequently directed the one toward the centre of the
figure, the other in a direction at right angles to
the first.

The exact or even approximate determination of
the figures produced is a complicated question, and
requires mathematical analysis for its solution, but a
general idea of their nature may nevertheless be easily
formed.   When the polariser and analyser are either
parallel or crossed, circular rings are formed, and
towards the outer parts of the field traces of the black
cross are seen, which grow stronger as we proceed
outwards from the centre—that is, towards the parts
where the rays are more oblique, and where the
polarisation more nearly approaches to rectilinear.
But in the centre, and near to it, where the polarisa-
tion is circular or nearly so, the effects will resemble
those produced by parallel rays, viz. the rays of
different colours will emerge plane-polarised in
different planes, and will be variously affected by the
angle between polariser and analyser.   In no position

can they all be extinguished, and consequently in the centre all traces of the black cross will disappear. It should, however, be added that when the quartz plate is very thin, say half a millimetre, the planes of polarisation of the different coloured rays are so little dispersed, that the analyser extinguishes them all more or less at once. On this account the arms of the black cross extend inwards with tolerable distinctness to the centre of the field.

When the planes of vibration of the polariser and the analyser are inclined at any other angle than 0° or 90°, the arms of the cross are less strongly marked, and the curvature of the rings becomes less uniform, increasing in the four points where they are crossed by the arms, and diminishing in the intermediate quadrants. When the angle between the planes of vibration is 45°, the rings assume a nearly square form, the corners of the square lying upon the lines which bisect internally and externally the angles between the planes. If the figures are produced with the analyser at 45° by two quartz plates of equal thickness, one right-handed, the other left, it will be found that the diagonals of the squares are at right angles to one another, the remains of the black cross occupying the same position in the field in both cases. (*See* Plate, Figs. 9 and 10.)

If two plates of the same thickness, the one right-handed and the other left, are placed one over the other, a beautiful effect, called from their discoverer Airy's spirals, is produced (*see* Plate, Figs. 11 and 12). In the centre of the field the rotatory powers of the plates neutralise one another, and a black cross com-

mences. As we proceed outwards, the arms of the cross cease to be black, and become tinged with red on one side and with blue on the other. At the same time they are bent round in a spiral form, in the direction of the hands of a watch if the first plate be right-handed, and in the opposite direction if the first plate be left-handed. These spirals intersect at intervals the circular rings ; the points of intersection lie in four rectangular directions, which terminate towards the outer margin of the field in four arms of a shadowy cross. The colours of the rings and spirals are more brilliant and better defined than in most other phenomena of chromatic polarisation.

A quartz plate cut parallel to the axis, when examined with convergent light, gives curves in the form of hyperbolas. These curves are wider in proportion to the thinness of the plate, but if the plate be thick enough to render the curves moderately fine, the colour becomes very faint. They may, however, be rendered distinct by using homogeneous light. The dark and light parts exchange positions when the analyser is turned through 90°. Two such plates with their axes at right angles to one another give coloured hyperbolas perfectly visible with the white light. Plates of Iceland spar exhibit similar phenomena, but the lines and curves are far more closely packed.

If the plate be cut in a direction inclined at 45° (or at any angle differing considerably from 0° or 90°) to the axis, the curves are approximately straight lines perpendicular to the principal section of the plate. Two such plates placed with their principal

planes at right angles to one another give straight lines bisecting the angle between the principal planes. On this principle Savart constructed the polariscope which bears his name. It consists of two such plates and an analyser, and forms a very delicate test of the presence of polarisation. The lines are of course always in the direction described above, and the delicacy of the test increases in proportion as their direction becomes more and more nearly parallel or perpendicular to the original plane of vibration.

Bi-axal crystals exhibit a more complicated system of rings and crosses, or brushes, as they may in this case be better termed. If such a crystal be cut in a
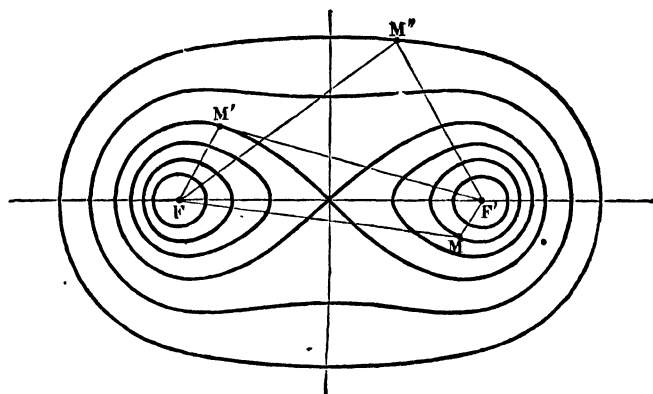
FIG. 26.

direction perpendicular to the line which bisects the angle between the two optic axes (or the middle line, as it is called), the extremity of each of the axes will be surrounded with rings similar to those described in the case of the uni-axal crystals. The larger rings,

however, are not strictly circles, but are distorted and drawn out towards one another; those which are larger still meet at a point midway between the centres and form a figure of 8 or lemniscata; beyond this they form curves less and less compressed towards the crossing point, and approximate more and more nearly to an oval. (*See* Fig. 26; also Plate, Figs. 3 and 4.)

The vibrations of the two rays emerging from any point of a bi-axal crystal are as follows :—Of the two rays produced by the double refraction of a bi-axal crystal neither follows the ordinary law of refraction ; but one does so more nearly than the other, and is on that account called for convenience the ordinary ray. And if through any point of the field of view we draw two lines to the points where the optic axes emerge, the directions of vibration of the two rays will be those of the bisectors of the angles made by the two lines. If, therefore, the crystal be so placed that the line joining the extremities of the two axes coincides with the plane of vibration of either polariser or analyser, it is not difficult to see that there will be a black cross passing through the centre of the field with one pair of arms in the line joining the extremities of the axes and the other pair at right angles to it. But if the plate be turned in its own plane round the central point, the points for which the vibrations are parallel or perpendicular to those of the polariser or analyser will no longer lie in straight lines passing through the centre, but will form two branches of a hyperbolic curve, passing respectively through the extremities of the optic axes.

If the analyser be turned round, the dark hyperbolic brushes, or the black cross, will undergo the changes analogous to those shown by the cross in the case of uni-axal crystals; but the most interesting effects are those seen when the polariser and analyser are crossed, and the crystal is turned in its own plane.

The angle between the optic axes in different kinds of crystals varies very much; in those where the angle is small it is easy to exhibit both at once in the field of view, but in others where the angle is large it is necessary to tilt the crystal so as to bring the two successively into view. In the latter case the crystal is sometimes cut in a direction perpendicular to one of the axes. The rings are then nearly circular, especially towards the centre, and in that respect they resemble those of a uni-axal crystal; but the character of the specimen can never be mistaken, because the rings are intersected by a black bar, or two arms in the same straight line, instead of by four arms at right angles to one another, as would have been the case if the crystal had been uni-axal. The following are the angles made by the optic axes in a few crystals :—

ORTHORHOMBIC CRYSTALS.

		Apparent Angle.	Actual Angle.
Aragonite . .	—	71° 31′	43° 32′
Cerceosite (Lead Carbonate) .	—	16° 54′	8° 7′
Chrysoberyl · .	+	84° 43′	45° 20′
Chrysolite . .	+	—	88° 54′
Topaz . .	+	120° to 97′	65° to 55
Anhydrite . .	+	71° 31′	43° 32′

		Apparent Angle.	Actual Angle.
Barytes . .	+	59° 6′	35° 4′
Potassio Sodium Tartrate (Rochelle Salt)	+	{76° red {56° violet	
Ammonium Tartrate		59° 35′ (D)	38° 2′
Ammonio Sodium Tartrate		{ 62° red {100° violet	{46° {70°
Nitre . . .	—	9° 17′	6° 10′

OBLIQUE CRYSTALS.

Diopside . .	+	111° 34′	58° 59′
Epidote . .	—	115° 20′ to 92° 4′	87° 49′ to 74°
Felspar Orthoclase)	—	{121° 6′ { 20° 45′	{69° 43′ {13° 34′
Sphene . .	+	{56° to 53° 30′ red {34° to 32° 30′ blue	
Dichroite . .	—	{Ceylon 125° 16′ {Haddam 60° 46′	70° 23° 37° 48′
Axinite . .	—	158°	71° 30′
Gypsum . .	+	max.	75° 40′
		varying with temperature.	
Borax . .	+	59° 8′	39° 14′
Mica . . .	+	0° to 75°	

$\frac{1}{2}$
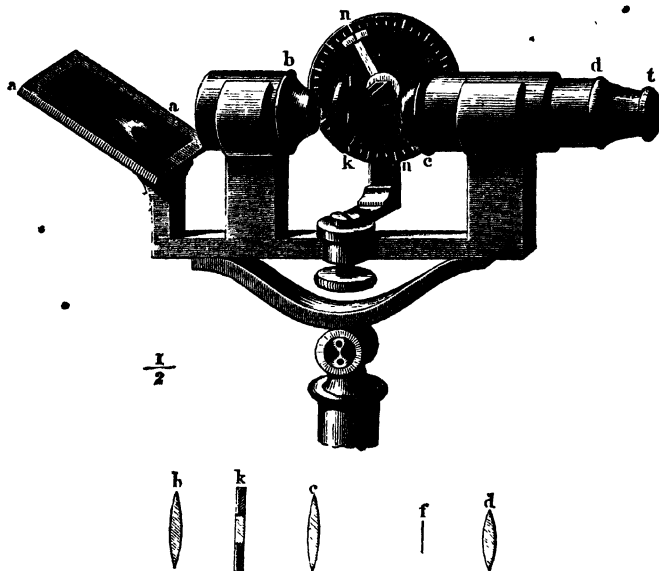
FIG. 27.
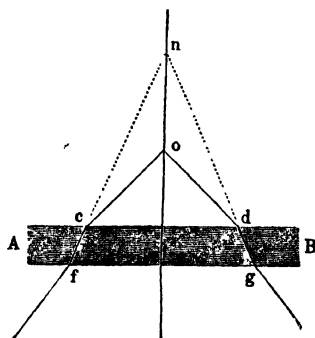
FIG. 28.

These angles are determined by placing the crystal for examination into an apparatus adapted to show the rings, and attaching it to an arm whereby the plate can be turned about an axis in its own plane. The axis is furnished with a circle divided into degrees and seconds, and an index. (*See* Fig. 27.) If this axis be horizontal, the plate is so placed that the line joining the centres of the two systems of rings is vertical, and the crystal is first turned so as to bring one centre into the centre of the field of view (usually marked by cross wires) ; the index is then read, and the crystal turned so as to bring the centre of the second system of rings to the centre of the field. The index is again read, and the difference of the two readings noted. This, however, gives not the true angle of the optic axis, but the apparent angle in air—that is, the angle between the rays as affected by refraction on emerging from the crystal. (*See* Fig. 27.) In the annexed figure (28) A B represents the crystal plate, *o* the position of the eye, and *f c*, *g d* the directions of the axes in the crystal ; the observed angle is *c o d*. But if the (mean) refractive index of the crystal is known, the true angle, *c n d*, may be calculated.

In bi-axal crystals the optic axes have, in the majority of cases, different angles for different colours, and where no limits are imposed on the direction of the first mean-line (the line bisecting the acute angle of the optic axes) by the conditions of symmetry of the crystallographic system, the mean-lines for different colours may be, and usually are, slightly inclined to each other, or, as it is termed, *dispersed.*

In the orthorhombic system the first mean-line for

all colours coincides in direction with one of the crys-
tallographic axes : the optic axes always lying in one
or other of the axial planes (viz. in a plane containing
the particular axis and one other crystallographic
axis). The optic axes for either the more or the less
refrangible rays may be the more widely separated :
it is rare for them not to present differences in this
respect.

In the clinorhombic (or oblique) system :

1. The first mean-line may be parallel to the per-
pendicular axis. The planes containing the optic
axes belonging to different colours will, in this case, all
intersect in this line. The angles of the optic axes may
vary for every colour. Borax is an example of this.

2. The first mean-line for each colour may lie in
the plane containing the oblique axes (the one plane
of symmetry) of the system. The planes containing
the optic axes (and, so, the second mean-lines) may
lie in this plane ; the mean-lines for different colours
being dispersed in this plane. Where the optic axes
for different colours have different degrees of di-
vergence, the two "eyes" of the figure are very dis-
similar, the one being larger than the other : where
the divergence is not great, the colour that is outside
in the one is inside in the other. Diopside, gypsum,
and sphene are examples of this variety. (*See* Plate
at end.)

3. And while the mean-lines lie in the plane of
symmetry, the planes of the optic axes for different
colours may be perpendicular to this plane (the second
mean-line therefore being parallel to the perpendicular
axis). This kind of dispersion, then, shows the "eyes"

for the red and blue, for example, dispersed on lines parallel to each other. Of this, adularia affords an example.

Lastly, in the anorthic system, the mean-lines for different colours may lie in different planes, as may also the optic axes which they bisect.

There are crystals of the orthorhombic and clino-rhombic systems, moreover, which present a very remarkable variety of dispersion. Thus in brookite chrysoberyl, some of the alkali bibasic salts of tartaric acid, ammonium thallate, magnesio-ammonium, chro-mate, &c., the optic axes for the two extremities of the spectrum lie in planes at right angles to one another, both passing through the same middle line. If the systems of rings be examined with light which has been so widely dispersed that the portion illumi-nating the field in any given position is practically monochromatic, and the position of the instrument shifted through the different parts of the spectrum (or, what is more convenient, if the different parts of the spectrum be successively thrown on the polariscope by means of a totally reflecting prism), the optic axes will be seen to draw gradually together until the figure closely resembles that of a uni-axal crystal ; after which the axes open out in a direction at right angles to the former, until they have attained their greatest expansion. This experiment requires a strong light, but it is instructive as showing the exact distribution of the optic axes for different rays.

In bi-axal crystals, probably in all cases, but notably in gypsum, the distribution of the optic axes varies with the temperature. When the crystal is

heated, the angle between the optic axes diminishes until the crystal appears uni-axal; with a further increase of temperature the axes again open out, but in a direction at right angles to the former. When the crystal is cooled, the axes generally resume their original directions. Sometimes, however, when the heating has been carried to a great degree, or has been continued for a long time, the axes never completely return to their normal condition; and in such a case the crystal may appear permanently uni-axal. Such an appearance, when permanent, has been considered a test of former heating; and this phenomenon, when presented by crystals found in a state of nature, may be taken as evidence that the rocks in which they have been formed have been subject to high temperatures.

In the production and examination of the rings hitherto described, we have used light which has been plane-polarised and plane-analysed; but there is nothing to prevent our polarising the light or analysing it circularly, or indeed doing both.

If a quarter-undulation plate be placed between the polariser and the crystal to be examined, with its axis inclined at 45° to the plane of original vibration, the light will fall upon the crystal in a state of circular polarisation; and as the polarisation will then have no reference to any particular plane of vibration, the black cross will disappear. A system of rings will be produced, but they will be discontinuous. At each quadrant, depending upon the position of the analyser, the rings will be broken, the portions in opposite quadrants being contracted or expanded, so that in

passing from one quadrant to the next the colours
pass into their complementaries.   If either the direc-
tion of the axis of the quarter-undulation plate be
changed from 45° on one side to 45° on the other side
of the plane of vibration of the polariser, or if the
crystal be changed for another of an opposite character
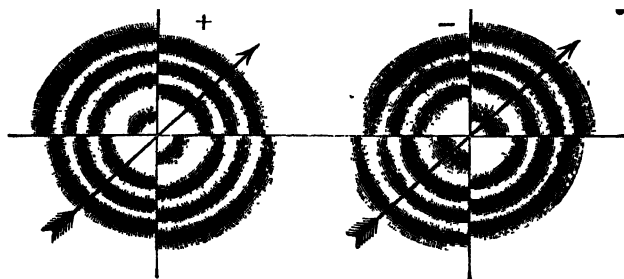
FIG. 29.

(*i.e.* negative for positive, or *vice versâ*), the quadrants
which were first contracted will be expanded, and
those which were first expanded will be contracted.
Hence for a given position of the quarter-undulation
plate the appearance of the rings will furnish a means
of determining the character of the crystal under
examination.

Similar effects are produced if the quarter-undula-
tion plate be placed between the crystal and the
analyser ; that is, if the light be analysed circularly.

In the case of bi-axal crystals under the action of
light polarised or analysed circularly, the black brushes
are wanting, but they are replaced by lines of the same

form, marking where the segments of the lemniscatas pass from given colours into their complementaries.

If the light be both polarised and analysed circularly, all trace of direction will have disappeared. In uni-axal crystals the rings will take the form of perfect circles without break of any kind; and in bi-axal they will exhibit complete lemniscatas.

• To pursue this matter one step further, suppose that, the arrangements remaining otherwise as before (viz, first, the polariser; secondly, a quarter-undulation plate with its axis at 45° to the principal plane of the polariser; thirdly, a uni-axal crystal; fourthly, a quarter-undulation plate with its axis parallel or perpendicular to the first; and, lastly, the analyser), the analyser be turned round; then in any position intermediate to 0° and 90° the rings will be contracted and extended in opposite quadrants until at 45° they are divided by two diagonals, on each side of which the colours are complementary. Beyond 45° the rings begin to coalesce, until at 90° the four quadrants coincide again. During this movement the centre has changed from bright to dark. If the motion of the analyser be reversed, the quadrants which before contracted now expand, and *vice versâ*. Again, if the crystal be replaced by another of an opposite character —say positive for negative—the effect on the quadrants of the rings will be reversed. This method of examination, therefore, affords a test of the character of a crystal.

A similar process applies to bi-axal crystals; but in this case the diagonals interrupting the rings are replaced by a pair of rectangular hyperbolas, on either

side of which the rings expand or contract, and the
effect is reversed by reversing the motion of the ana-
lyser, or by replacing a positive by a negative crystal.
The test experiment may then be made by turning
the analyser slightly to the right or left, and observing
whether the rings appear to advance to, or recede
from, one another in the centre of the field.   In
particular if, the polariser and analyser being parallel,
the first plate have its axis in a N.E. direction to a
person looking through the analyser, the second plate
with its axis at right angles to the former, and the
crystal be so placed that the line joining the optic axes
by N.S., then on turning the analyser to the right the
rings will advance towards one another if the crystal be
negative, and recede if it be positive.

It was explained in a former chapter that the
coloured bands and rings shown by white light, when
polarised and transmitted through crystals, fade, and
cease to be visible when the retardation of the rays,
due to the thickness of crystal traversed, is large.
The feebleness of tint and confusion of definition
arises from the overlapping of figures of different
colours.   But when monochromatic light is used no
mixture of colour can take place, and the bands and
rings remain perfectly defined, even when the thick-
ness of the crystal is considerable.   The more com-
plicated figures produced by two plates of crystal are
still more liable to obliteration ; and the use of mono-
chromatic light is in this case even more important
for maintaining the integrity of the phenomena.

One essential requisite for bringing out the figures
in question is purity of colour in the light employed.

On this account the ordinary method of absorption by coloured media fails ; and it is only by the use of a monochromatic source of light that a satisfactory result can be obtained. For eye observations, a spirit lamp, sometimes with the addition of a little salt, suffices ; but the illumination from this source is insufficient for projection on a screen.   If, however, the lime in an oxy-hydrogen lamp be replaced by a mixture formed by melting borax, with the addition of some small pieces of hard German glass, and a bead of this substance be placed in a small platinum cup, so fixed that the jet of mixed gases could play upon it at a distance of about three-sixteenths of an inch, a good monochromatic light is obtained. This arrangement serves perfectly for laboratory and experimental work ; but for projection on a large scale a still more powerful source of light is required. The only burner adapted to lecture purposes is due to a suggestion of Professor Dewar. The burner consists of an oxy-hydrogen jet, with the addition to the hydrogen tube of a chamber containing metallic sodium. The metal is volatilised by a Bunsen's burner placed below it ; so that the hydrogen emerges charged with sodium vapour. The result is a bright monochromatic light.

If convergent or divergent light is used, uniaxal crystals (with which alone we are here concerned), when cut perpendicularly to their axes, show the well-known systems of isochromatic rings and dark brushes. When cut at other angles, they show portions of the same systems. But when two such plates are used in combination, theory indicates the presence of some

secondary phenomena, of which only very small por-
tions are visible with white light ; but with mono-
chromatic light the configuration may be traced
throughout the entire field of view.

In order to examine the mode of formation of
these secondary figures, it will be simplest to begin
with some thick plates and moderately convergent
light, by which means the details may be brought out
on a sufficiently large scale. For this purpose may
be taken pairs of plates, cut respectively at 67° 30′,
45°, 22° 30′ to the axis. These, and more particularly
the second, are used for producing the well-known
Savart's bands. If convergent light be made to
traverse any of these plates, some portion of the
rings are produced. The proportion of the ring
system contained in the field of view depends upon
the convergence of the rays, and the distance of the
centre of the rings from that of the field upon the
inclination to the axis at which the plate is cut.

If a pair of these plates be used, and the principal
plane of one, originally coincident with that of the
other, is made to turn gradually through 180°, then
with white light a series of coloured bands is pro-
duced. The distance between the bands increases,
from a minimum when they are first visible, inde-
finitely as the angle of turning approaches 180° ;
while the brilliancy of their tints and accuracy of
definition attains its maximum at 90°. With mono-
chromatic light, when the angle between the principal
planes of the plates is 0°, the vibrations will traverse
the second plate in the same manner as they traversed
the first ; and the retardation between the ordinary

and the extraordinary rays will be double of that due to each plate singly. The two plates will thus act as a plate of double thickness, and the number of bands (portions of rings) visible with one plate, will be doubled. When one plate is turned as before in front of the other, and the angle between their principal planes is gradually increased, the rings due to one plate cross those due to the other; and the intensity of illumination at the overlapping parts will depend upon the angle at which the rings cross one another. Bearing in mind the fact that at every point of the field the polarisation of one ray is parallel, and that of the other perpendicular to the direction of the ring, it is seen that when the rings cross at 90°, the ray which traversed the first plate as an ordinary, will traverse the second as an extraordinary ray; and consequently the retardation due to the two plates together will be the difference of that due to each alone. The result will therefore be similar to that produced by the two wedges when their refracting edges are at right angles; and the field will be crossed by diagonal straight lines.

For all other angles of crossing, including the case particularised above, the following will take place. Each ray which enters the first plate will emerge as two rays polarised, the one radially, the other tangentially with respect to the rings, and with a certain retardation. On entering the second plate, the vibrations of each of these rays will be again resolved radially and tangentially with respect to the rings due to the second plate. Each of the new components, whether radial or tangential, will therefore

consist of two parts, generally of different intensities, one of them having been retarded behind the other in their passage through the first plate. The two parts of each tangentially vibrating ray will partially interfere; and so likewise will the two parts of each radially vibrating ray. In consequence of this interference, the two rays emerging from the second plate will in general be of different intensities, and one of them will be retarded behind the other in their passage through that plate. Finally all the vibrations will again be resolved into one plane by the analyser; and when so resolved they will in general partially interfere. This partial interference will cause the dark rings due to the first plate to be broken with patches of partial brightness, and the bright rings with patches of partial darkness, where the rings of one system cross those of the other. The general effect is in many cases that of a diaper pattern over the field of view.

The distribution of these interruptions and the nature of the secondary figures which they form, will depend upon the curvature and angle of crossing of the rings at each point of the field. The mathematical formulæ, which give the details of the illumination, present no difficulty beyond tediousness of calculation. And it will be sufficient here to say that, when carried only to a first approximation (with respect to the angle of incidence of the rays) they indicate, for the secondary figures, a series of straight lines alternately dark and bright, known as " Savart's bands." When carried to a second approximation the formulæ indicate that in the neighbourhood of

the ring-centres, the secondary figures will be conic
sections. When the principal planes of the crystals
(planes containing the axis and the normal to the
plate) are at 180° to one another, the conic sections
are central. In that case, the expression for the
square of one of the principal axes of the curve is a
cubic in the line of the angle at which the crystal has
been cut. This expression when equated to zero
must, by the theory of equations, have one real root;
in other words, it will vanish for one particular value
of the angle, and be negative for greater values and
positive for less values of the angle. If, therefore,
the crystals be cut at an angle to the axis smaller
than the angle given by the cubic equation, they will,
when placed with the axes inclined to opposite sides
of the field, show hyperbolas for the secondary figures;
when cut at a certain angle (about 59° 50′ in the case
of quartz), the figures will be straight lines parallel
to the line joining the ring-centres; and when cut at
a greater angle, the figures will be ellipses, approxi-
mating to circles as the angle of section approaches
to 90°.

But leaving aside the mathematical aspect of the
question, the principal interest of the method of
monochromatic light consists in the simplicity of the
results, and in the opportunity which it affords of
examining in detail all the effects due to two plates
of crystal. It enables us in fact to follow the pecu-
liarities of the secondary figures throughout the entire
field of view, and to trace by a continuous process the
modifications which these figures undergo when the
relative positions of the crystals are changed.

Many of these effects are best seen with the optical arrangement usually employed for showing the crystal rings. Take, for instance, four pairs of quartz plates cut respectively at 45°, 59° 50′, 67° 30′, 90°, to the axis; the plates of each pair having their principal planes at 180° to one another. The first of these shows, in the region about the line joining the centres of the ring systems, a series of ellipses for the secondary figures; the second shows straight lines; the third oblique hyperbolas; the fourth rectangular hyperbolas. With a view, however, of exhibiting not only these the more important, but also all intermediate phases of the phenomena, may be taken two curved sections of quartz, each forming nearly a quadrant, and cut at one end perpendicular, at the other nearly parallel to the axis. By placing these end to end, with the axes at opposite ends, and sliding one over the other, all the phases of the secondary figures can be shown in succession. And if the point midway between the axes be kept in the centre of the field of view, the figures will be symmetrical; otherwise unsymmetrical. Beginning with the ends, which are perpendicular to the axis, together in the field of view, and causing the quadrants to slide at the same rate in opposite directions, the secondary circles will be seen elongating into ellipses, the ellipses stretching out until they pass into parallel straight lines; and lastly, these lines contracting towards the centre of the field and diverging towards the sides, until they form hyperbolas, the obliquity of which gradually diminishes as we approach that part of the section which is parallel to the axis.

The case of two quartz plates cut at an angle of 67° 30' to the axis, whereof one is free to revolve in front of the other, gives rise to some interesting transformations of the secondary figures. When the principal planes of the crystals are coincident, the field generally shows rings double in number to those due to one plate. But towards the side away from which the axes are directed the rays are more nearly parallel, while towards the opposite side the rays are more nearly perpendicular to the axis. Hence, towards the first side, the rings will approximate more nearly to circles, and towards the other they will show indications of becoming branches of hyperbolas. As the principal planes are turned round in opposite directions from their initial position, the secondary figures begin to appear. At 45° discontinuous bands with hyperbolic curvature towards the part of the field most distant from the ring-centres are seen. At 90° these bands become continuous and sharply defined, while towards the ring-centres a portion of the ellipses may be observed entering the field. Beyond 90° the hyperbolic branches leave the field, and the rectilinear part of the bands is replaced by the diaper pattern described in a former part of this lecture; while the ellipses are gradually elongated to parallel straight lines, and then are converted into hyperbolas. At 180° the hyperbolas occupy the centre of the field.

Many similar experiments may be made with biaxal crystals; but it would exceed our present limits to describe them here.

CHAPTER IX.

COMPOSITION OF COLOURS BY POLARISED LIGHT.[1]

THE results of combining two or more colours of the spectrum have been studied by Helmholtz, Clerk-Maxwell, Lord Rayleigh, and others. And the combinations have been effected sometimes by causing two spectra at right angles to one another to overlap, and sometimes by bringing images of various parts of a spectrum simultaneously upon the retina. Latterly also W. von Bezold has successfully applied the method of binocular combination to the same problem (*Poggendorff*, Jubelband, p. 585). Some effects, approximating more or less to these, may be produced by chromatic polarisation.

Complementary Colours.—First, as regards complementary colours. If we use a Nicol's prism N as polariser, a plate of quartz Q cut perpendicularly to the axis, and a double-image prism P as analyser, we shall, as is well known, obtain two images whose colours are complementary. If we analyse these images with a

[1] Extracted from the Proceedings of the Royal Society, No. 153, 1874. See also Proceedings of the Royal Institution, 1874.

prism, we shall find, when the quartz is of suitable
thickness, that each spectrum contains a dark band,
indicating the extinction if a certain narrow portion
of its length. These bands will simultaneously shift
their position when the Nicol·N is turned round.
Now, since the colours remaining in each spectrum are
complementary to those in the other, and the portion
of the spectrum extinguished in each is complementary
to that which remains, it follows that the portion ex-
tinguished in one spectrum is complementary to that
extinguished in the other. And in order to determine
what portion of the spectrum is complementary to the
portion suppressed by a band in any position we
please, we have only to turn the Nicol N until the band
in one spectrum occupies the position in question, and
then to observe the position of the band in the other
spectrum. The combinations considered in former
experiments are those of simple colours ; the present
combinations are those of mixed tints, viz. of the
parts of the spectrum suppressed in the bands. But
the mixture consists of a prevailing colour correspond-
ing to the centre of the band, together with a slight
admixture of the spectral colours immediately adjacent
to it on each side.

The following results given by Helmholtz may be
approximately verified :—

<div align="center">

Complementary Colours.

Red.	Orange.
Yellow.	Yellow-green.
Green-blue.	Cyanic-blue.
Indigo-blue.	Violet.

</div>

'When in one spectrum the band enters the green, in the other a band will be seen on the outer margin of the red, and a second at the opposite end of the violet ; showing that to the green there does not correspond one complementary colour, but a mixture of violet and red, *i.e.* a reddish purple.

Combination of two Colours.—Next, as to the combination of two parts of the spectrum, or of the tints which represent those parts. If, in addition to the apparatus described above, we use a second quartz plate Q_1 and a second double-image prism P_1, we shall form four images—say O O, O E, E O, E E. And if A, A′ be the complementary tints extinguished by the first combination Q P alone, and B, B′ those extinguished by the second $Q_1 P_1$ alone, then it will be found that the following pairs of tints are extinguished in the various images.

Image.	Tints extinguished.
O O	B, A
O E	B′, A′
E O	B′, A
E E	B, A′

It is to be noticed that in the image O E the combination $Q_1 P_1$ has extinguished the tint B′ instead of B, because the vibrations in the image E were perpendicular to those in the image O formed by the combination Q P. A similar remark applies to the image E E.

The total number of tints which can be produced

by this double combination Q P, Q₁ P₁ is as fol-.
lows :—

> 4 single images
> 6 overlaps of two
> 4 overlaps of three
> 1 overlap of four
>
> ——
>
> Total 15

Collateral Combinations.— The tints extinguished
in the overlap O O + E O will be B, A, B′, A ; but since
B and B′ are complementary, their suppression will not
affect the resulting tint except as to intensity, and the
overlap will be effectively deprived of A alone ; in
other words, it will be of the same tint as the image
O would be if the combination Q₁ P₁ were removed.
Similarly, the overlap O E + E E will be deprived
effectually of A′ alone ; in other words, it will be of
the same tint as E, if Q₁ P₁ were removed.   If, there-
fore, the Nicol N be turned round, these two overlaps
will behave in respect of colour exactly as did the
images O and E, when Q P was alone used. ' We may,
in fact, form a table thus :—

Image.	Colours extinguished.
O O + E O	$B + A + B′ + A = B + B′ + A = A$
O E + E E	$B′ + A′ + B + A′ = B + B′ + A′ = A′$

And since the tints B, B′ have disappeared from each
of these formulæ, it follows that the second analyser P
may be turned round in any direction without altering
the tints of the overlaps in question.

"In like manner we may form the table—

$$O\,O + E\,E \qquad B + A + B + A' = B + A + A' = B$$
$$O\,E + E\,O \qquad B' + A' + B' + A = B' + A + A' = B'$$

Hence, if the Nicol N be turned round, these overlaps will retain their tints ; while if the analyser P_1 be turned, their tints will vary, although always remaining complementary to one another.

There remains the other pair of overlaps, viz. :—

$$O\,O + O\,E \qquad B + A + B' + A$$
$$E\,O + E\,E \qquad B' + A + B + A'$$

Each of these is deprived of the pair of complementaries A, A', B, B' ; and therefore each, as it would seem, ought to appear white of low illumination, *i.e.* grey. This effect is, however, partially masked by the fact that the dark bands are not sharply defined like the Fraunhofer lines, but have a core of minimum or zero illumination, and are shaded off gradually on either side until a short distance from the core the colours appear in their full intensity. Suppose, for instance, that B' and A' were bright tints, the tints resulting from their suppression would be bright. On the other hand, the complementary tints A and B would be generally dim, and the image B + A bright, and the overlap B + A + B' + A' would have as its predominating tint that of B + A. And similarly in other cases.

There are two cases worth remarking in detail, viz., first that in which

$$B = A', \; B' = A$$

i.e. when the same tints are extinguished by the combination Q P and by $Q_1 P_1$. This may be verified by either using two similar quartz plates Q, Q_1, or by so turning the prism P_1, that the combination $Q_1 P_1$ used alone shall give the same complementary tints as Q P when used alone. In this case the images have for their formulæ the following :—

O O	O E	E O	E E
A + A'	A + A'	2A	2A'

in other words, O O and E O will show similar tints, and E O, E E complementary. A similar result will ensue if B = A B' = A'.

Again, even when neither of the foregoing conditions are fulfilled, we may still, owing to the breadth of the interference bands, have such an effect produced that sensibly to the eye

$$B + A = B' + A''$$

and in that case

$$B' + A = B + A - A' + A$$
$$= B + A' + 2A - 2A'$$

which imply that the images O O and O E may have the same tint; but that E O and E E need not on that account be complementary. They will differ in tint in this, that E E, having lost the same tints as E O, will have lost also the tint A, and will have received besides the addition of two measures of the tint A'.

Effect of Combination of two Colours.—A similar train of reasoning might be applied to the triple over-

laps. But the main interest of these parts of the figure consists in this, that each of the triple overlaps is complementary to the fourth single image; since the re-combination of all four must reproduce white light. Hence the tint of each triple overlap is the same to the eye as the mixture of the two tints suppressed in the remaining image. And since by suitably turning the Nicol N or the prism P_1, or both, we can give any required position to the two bands of extinction, we have the means of exhibiting to the eye the results of the mixture of tints due to any two bands at pleasure.

Effect of Combinations of three Colours.—A further step may be made in the combination of colours by using a third quartz Q_2 and a third double-image prism P_2, which will give rise to eight images. And if C C′ be the complementaries extinguished by the combination Q_2 P_2, the formulæ for the eight images may be thus written :—

O O O	C + B + A
O O E	C + B′ + A′
O E O	C′ + B′ + A
O E E	C′ + B + A′
E O O	C′ + B + A
E O E	C′ + B′ + A′
E E O	C + B′ + A
E E E	C + B + A′

The total number of combinations of tint given by the compartments of the complete figure will be—

$$\frac{8}{1} = 8 \text{ single images.}$$

$$\frac{8 \cdot 7}{1 \cdot 2} = 28 \text{ overlaps of two.}$$

$$\frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3} = 56 \quad \text{,,} \quad \text{,,} \quad \text{three.}$$

$$\frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4} = 70 \quad \text{,,} \quad \text{,,} \quad \text{four.}$$

$$\frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3} = 56 \quad \text{,,} \quad \text{,,} \quad \text{five.}$$

$$\frac{8 \cdot 7}{1 \cdot 2} = 28 \quad \text{,,} \quad \text{,,} \quad \text{six.}$$

$$\frac{8}{1} = 8 \quad \text{,,} \quad \text{,,} \quad \text{seven.}$$

$$1 = 1 \quad \text{,,} \quad \text{,,} \quad \text{eight.}$$

Total . . 255

The most interesting features of the figure consists in this, that the subjoined pairs are complementary to one another, viz. :—

O O O	E O E
$C + B + A$	$C' + B' + A'$
E O O	O O E
$C' + B + A$	$C + B' + A'$
E E O	O E E
$C + B' + A$	$C' + B + A'$
E E E	O E O
$C + B + A'$	$C' + B' + A$

And if the prisms P, P_1, P_2 are so arranged that the separations due to them respectively are directed parallel to the sides of an equilateral triangle, the images will be disposed thus :—

```
              O E O      O O O
     E E O    E O O      O E E      O O E
              E E E      E O E
```

The complementary pairs can then be read off, two horizontally and two vertically, by taking alternate pairs, one in each of the two vertical, and two in the one horizontal row. And each image will then represent the mixture of the three tints suppressed in the complementary image.

Low-tint Colours.—A slight modification of the arrangements above described furnishes an illustration of the conclusions stated by Helmholtz, viz. that the low-tint colours (*couleurs dégradées*), such as russet, brown, olive-green, peacock-blue, &c., are the result of relatively low illumination. He mentioned that he obtained these effects by diminishing the intensity of the light in the colours to be examined, and by at the same time maintaining a brilliantly illuminated patch in an adjoining part of the field of view. If, therefore, we use the combination N, Q, P, P_1 (*i.e.* if we remove the second quartz plate), we can, by turning the prism round, diminish to any required extent the intensity of the light in one pair of the complementary images, and at the same time increase that in the other pair. This is equivalent to the conditions of Helmholtz's experiments ; and the tints in question will be found to be produced.

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